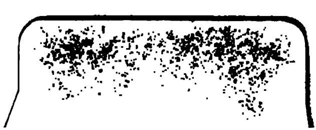

ALGEBRAICAL EXERCISES

PROGRESSIVELY ARRANGED

BY

THE REV. C. A. JONES, M.A.

LATE CHAPLAIN OF ST JOHN'S COLLEGE, CAMBRIDGE,

AND

C. H. H. CHEYNE, M.A.

ST JOHN'S COLLEGE, CAMBRIDGE, AUTHOR OF "AN ELEMENTARY TREATISE ON THE PLANETARY THEORY";

MATHEMATICAL MASTERS OF WESTMINSTER SCHOOL.

Cambridge and London:

MACMILLAN AND CO.

1865.

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS

PREFACE.

This little book is intended to meet a difficulty which is probably felt more or less by all engaged in teaching Algebra to beginners. It is that while new ideas are being acquired, old ones are forgotten. In the belief that constant practice is the only remedy for this, the present series of miscellaneous exercises has been prepared. Their peculiarity consists in this, that though miscellaneous, they are yet progressive, and may be used by the pupil almost from the very commencement of his studies. They are not intended to supersede the systematically arranged examples to be found in ordinary treatises on Algebra, but rather to supplement them.

The exercises have been selected mainly from recent School and College Examination Papers, and have been arranged on the following plan: the first forty, commencing with division, take in gradually

such portions of the subject as are usually studied before Surds: in Exercises XLI. to LV. Surds are introduced; in LVI. to LXX. Ratio, Proportion, Variation, the Progressions, and Indeterminate Equations; and in LXXI. to C. Permutations and Combinations, the Binomial Theorem, and Notation. Lines have been placed to mark these divisions of the subject and it will be seen that the Exercises immediately preceding them are considerably harder than some few of those which follow. This arrangement has been made to meet the wants of pupils of different abilities. The book being mainly intended for Schools and Junior Students, the higher parts of Algebra have not been included.

Every effort has been made to insure accuracy but information with regard to errors which may be detected, and suggestions for the improvement of the Exercises will be thankfully received.

I, DEAN'S YARD, WESTMINSTER,
July 25, 1865.

ALGEBRAICAL EXERCISES.

EXERCISE I.

1. If $a=1, b=2, c=3$, find the numerical value of

$$\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}.$$

2. Add together

$$5x+3y-2z,\ -3x+2y-5z,\ 2x-5y+3z, \text{ and } -4x+4z.$$

From $x-y+z$ take $-x-y-z$.

3. Multiply $3x+2y-z$ by $x-2y+3z$, and prove your result by division.

4. Divide a^3-x^3 by $a-x$, and prove your result by multiplication.

EXERCISE II.

1. Add together

$$-7x^3+5x^2y-8xyz+2xz^2,$$

$$-7y^3+5y^2z+4xyz-x^2z,$$

$$-3x^2y+2x^2z-6xyz+11xz^2,$$

and $$x^3-x^2y+x^2z-xyz+xy^2-xz^2+yz^2;$$

also from the sum subtract

$$2x^2y-3y^2z+4xz^2.$$

2. Simplify the expressions

(i) $a+(b-c)-(a-c)$,

(ii) $(7x^2-5xy+6y^2)-(2x^2+3xy-y^2)$
$-(3x^2-2xy)+(4xy-5y^2)$.

3. Multiply $2x^2-3y^2-xy$ by $5x^2-2xy$,
and divide $6x^5-4x^4-19x^3+23x^2-13x+3$ by $3x^2-2x+1$.

4. Divide
$a^4+b^4+2a^2b^2-2c^2d^2-c^4-d^4$ by $a^2+b^2-c^2-d^2$.

Exercise III.

1. Simplify the expression

$$5x-\{3x-(2x+3)\},$$

and find its value when $x=-\frac{3}{4}$.

2. From $-5a^2+3b^2+c^2-7ab+8bc-3ca$
take $-7a^2-3b^2+6c^2+9ab-4bc-5ca$.

3. Multiply x^2-px+q^2 by x^2+px-q^2,
and divide $a^4+a^2b^2+b^4$ by a^2-ab+b^2.

4. Solve the equations

(i) $5x-7=2x+2$,

(ii) $2(x-1)+3(x-2)=x$.

Exercise IV.

1. Simplify $3a-[2a-2\{a-(a-1)\}+2]$.

Add together $a^2-b^2+\frac{c^2}{6}$, $a^2+b^2-\frac{c^2}{3}$, $b^2-a^2+\frac{c^2}{6}$, and *from the sum* subtract $a^2+b^2-c^2$.

2. Multiply $3a^2+2ab+b^2$ by $a^2-2ab+3b^2$, and prove your result by division.

3. Divide $x^4-x^3y-xy^3+y^4$ by x^2+xy+y^2.

4. Solve the equations

(i) $\frac{x}{3}+\frac{x}{7}=x-11$,

(ii) $\frac{x+1}{2}+\frac{x+2}{3}=14+\frac{5-x}{4}$.

Exercise V.

1. If $a=1$, $b=2$, $c=3$, $d=4$, find the value of

$$bc^2d-c^2bd+abcd+a^2(3bc-4b^2+5c^2)-a^3(7b-8d).$$

2. Add together

$$5ax-3by+4cz,\ -2ax+4by-3cz,$$
$$-ax+7by-cz,\ 9ax-11by+10cz;$$

and from the sum subtract $-ax-by-cz$.

3. Multiply $2x^3-5x^2+4x-1$ by $3x^2-x+2$, and divide $6x^4+5x^3+6x^2-17x+6$ by $6x^2-7x+2$.

4. Solve the equations

(i) $(x-1)(x-2)+(x-1)(x-3)=2(x-2)(x-3)$,

(ii) $\frac{5-3x}{4}+\frac{5x}{3}=\frac{3}{2}-\frac{3-5x}{3}$.

Exercise VI.

1. Simplify

$$3\,[4x-5-2\,(3x-4)+5\,\{2x-3-(2x-7\,\overline{x-5})\}]$$

1—2

2. Multiply $x^2+y^2+z^2-yz-zx-xy$ by $x+y+z$.

3. Divide $a^4+a^2b^2+b^4$ by a^2+ab+b^2,

and x^5+y^5 by $x+y$.

4. Solve the equations

(i) $\frac{x}{2}+\frac{x}{3}-\frac{x}{4}+\frac{x}{5}=7\frac{5}{6}$,

(ii) $\frac{3x-5}{4}-\frac{7x+9}{16}+\frac{8x+19}{8}+8\frac{5}{8}=0$.

Exercise VII.

1. Simplify

$$3x-[y-\{x+(y-3a)\}],$$

and find the value of $(\sqrt{x^2+y^2}+z)\times(\sqrt{x^2+y^2}-z)$ when

$$x=8,\ y=6,\ z=4.$$

2. Add together

$$\frac{a^2}{2}-\frac{b^2}{3}+\frac{c^2}{4},\ \frac{b^2}{2}-\frac{c^2}{3}+\frac{a^2}{4},\ \frac{c^2}{2}+\frac{a^2}{3}+\frac{b^2}{4},$$

and from the sum take $c^2-a^2+\frac{b^2}{2}$.

3. Multiply $x^2-2xy+y^2$ by $x^2+2xy+y^2$, and find the continued product of

$$x+y,\ x-y,\ x^2+xy+y^2, \text{ and } x^2-xy+y^2.$$

4. Divide $\frac{x^3}{125}+27$ by $\frac{x}{5}+3$, and $a+2x$ by $a-2x$ to five terms.

5. Solve the equations

(i) $\frac{x-1}{2}-\frac{x-2}{3}=\frac{x-3}{4}$,

(ii) $(x+1)(x-2)=(x-3)(x+4)$.

Exercise VIII.

1. Add together

$$a(a-b+c+d),\ \ b(a+b-c+d),$$
$$c(a+b+c-d),\ \text{and}\ d(-a+b+c+d).$$

From $\dfrac{a+b}{2}$ take $\dfrac{a-b}{2}$.

2. Simplify the expressions

(i) $x-[\{2x-\overline{x-3(2x+y)}\}-3(2x+y)]$,

(ii) $3\{a-2(b-\overline{c+d})\}$,

and find the numerical value of the latter when $a=0$, and $b=c+d$.

3. Multiply $x+\frac{1}{2}y-2$ by $\frac{1}{4}x+3y$, and shew that

$$(x+2)(2x+1)-(x-2)(2x-1)=10x.$$

4. Divide $a^3+b^3+c^3-3abc$ by $a+b+c$.

5. Solve the equations

(i) $2(x+3)-3(x-4)+4(x+5)=38$,

(ii) $\dfrac{3x+1}{13}+\dfrac{2x-5}{3}=\dfrac{4x-1}{5}+\dfrac{2-x}{2}$.

Exercise IX.

1. Find the value of $(a-b)^2+(b-c)^2+(c-a)^2$ when

$$a=1,\ b=2,\ c=3,$$

and of $\dfrac{x-1}{x+1}+\dfrac{x+3}{x-3}-2\,\dfrac{x+2}{x-2}$ when $x=5$.

2. Simplify

$$3(a^2-b^2)-[2a^2-2\{b^2+ab+b(b-a+b)\}].$$

3. Multiply $a^2+4ax+5x^2$ by $a^2-3ax-2x^2$;

and $a^{3r}+a^{2r}b^p+a^rb^{2p}+b^{3p}$ by a^r-b^p.

4. Divide $\frac{2}{5}a^4x-\frac{136}{75}a^3x^2+\frac{8}{5}a^2x^3+\frac{3}{10}ax^4-x^5$

by $\frac{2}{3}a^3-\frac{4}{5}a^2x+\frac{1}{2}x^3$.

5. Solve the equations

(i) $\frac{3x}{2}-7+4x=\frac{9x}{2}-5$,

(ii) $\frac{bx}{a}-\frac{d}{c}=\frac{a}{b}-\frac{cx}{d}$.

Exercise X.

1. Find the value of

$$\{(a-b)(c+d)-(a-c)(b-d)\}(a-d)$$

when $a=4$, $b=3$, $c=2$, $d=1$.

2. Simplify the expression

$$5a-7(b-c)-[6a-(3b+2c)+4c-\{2a-(b+2c-a)\}],$$

and subtract $10a-9b$ from the sum of $5a-6b$ and $9a+10b$.

3. Shew that

$$(a-b)(a+b-c)+(b-c)(b+c-a)+(c-a)(c+a-b)=0,$$

and find the continued product of

x^2-y^2, x^2+y^2, x^4+y^4, and x^8+y^8.

4. Divide

$$a^8+a^6b^2+a^4b^4+a^2b^6+b^8 \text{ by } a^4+a^3b+a^2b^2+ab^3+b^4,$$

$$\text{and } x^4-\frac{1}{x^4} \text{ by } x-\frac{1}{x}.$$

5. Solve the equations

(i) $\dfrac{x-1}{7}+\dfrac{23-x}{5}=7-\dfrac{4+x}{4}$,

(ii) $2x-\{(2x+1)-(3x-6)\}=35$
$\qquad -3\,[2x-\{10-(2x-1)\}]$.

Exercise XI.

1. Simplify the following expressions

(i) $\{2x+y-(x+2y)\}\times\{3x-2y-(2x-3y)\}$,

(ii) $\left(1-\dfrac{a}{b}\right)b+\left(1-\dfrac{b}{c}\right)c+\left(1-\dfrac{c}{a}\right)a$,

and find the value of $1+\dfrac{1}{1-\dfrac{1}{x+1}}$ when $x=2$.

2. Add together

$$3a+5b-c,\ 2a-4b+c,\ \frac{b}{2}-a+\frac{c}{3},$$

and from the sum subtract $\dfrac{a}{2}-\dfrac{b}{2}-\dfrac{c}{3}$.

3. Multiply $3x^3+4x^2y-xy^2+4y^3$ by $x-2y$,

$$\text{and } x^{2r}-x^ry^s+\frac{y^{2s}}{4} \text{ by } x^{2r}+x^ry^s+\frac{y^{2s}}{4}.$$

Prove the result of the latter by division.

4. Divide

a^4-81 by $a-3$, and a^2 by a^2-x^2 to 4 terms.

5. Solve the equations

(i) $5x-\dfrac{2x-1}{3}+1=3x+\dfrac{x+2}{2}+7$,

(ii) $\dfrac{3x-\frac{1}{2}}{4}=2\left(x-\dfrac{5}{4}\right)$.

Exercise XII.

1. Simplify the expression

$$2\left[x-3\left\{x-4\left(x-\overline{\tfrac{5}{6}x-a}\right)\right\}\right].$$

Also add

$(\overline{a-b}\,x-\overline{b-c}\,y)-(\overline{a+b}\,x+\overline{b+c}\,y)$ to $ax+cy+b\,(x+y)$.

2. If $a=2$, find the numerical value of

$$a^{2a-1}+2a^{a-1}+(2a-1)^a.$$

3. The product of two algebraical expressions is

$$4a^2b^2+2\,(3a^4-2b^4)-ab\,(5a^2-11b^2),$$

and one of them is $3a^2+2ab-b^2$; find the other.

4. Shew that

$$\frac{x^6-y^6}{x^3-y^3}=(x+y)\,(x^2-xy+y^2).$$

5. Solve the equations

(i) $\frac{2}{3}(x-8)+\frac{3}{4}(x-9)-\frac{5}{6}(x-11)=7-\frac{3}{8}(x-17)$,

(ii) $\frac{3x-7}{4x+2}=\frac{3x-14}{4x-13}$.

Exercise XIII.

1. If $a=5$, $b=3$, $c=1$, find the numerical values of

(i) $\frac{a^3-b^3}{a^2+ab+b^2}+\frac{2a+5b}{2b-c}+\frac{a^2-c^2}{a+2b+c}$,

(ii) $\sqrt{5ab^2}+\sqrt[3]{9bc}-2\sqrt[4]{3a+b-2c}$.

2. Add together

$3a^3+a^2b-2ab^2+b^3$, $3ab^2-2a^2b+a^3$, $a^2b-ab^2+3b^3$,
and subtract half the sum from

$$3a^3+a^2b^2+3b^3.$$

3. Multiply $\frac{x^2}{2}-\frac{x}{3}+1$ by $\frac{x^2}{3}+\frac{x}{2}-1$, and find the square of $x^2+2ax-3a^2$.

4. Divide $a^2y^3-b(a^2+b)y+ab^2$ by $ay-b$,

and $\frac{3}{4}x^5-4x^4+\frac{77}{8}x^3-\frac{43}{4}x^2-\frac{33}{4}x+27$ by $\frac{x^2}{2}-x+3$.

5. Solve the following equations

(i) $1\frac{1}{3}-\frac{1}{3}(3x-2)=\frac{1}{2}(2-x)$,

(ii) $\frac{12x-2}{6}-(x+2)=\frac{18-4x}{3}$.

6. A boy is one-third the age of his father, and has a brother one-sixth of his own age; the ages of all these amount to 50 years. Find the age of each.

EXERCISE XIV.

1. Multiply together

$$x^2-x+1,\ x^2+x+1,\ \text{and}\ x^4-x^2+1,$$

and divide $x^{16}+x^8+1$ by the continued product.

2. Find the cube of $x-2y+3z$, and the fourth power of $2a^2-3ax$.

3. Find the values of

$$\sqrt{\frac{441a^{20}b^{10}}{100x^{14}y^6}},\qquad \sqrt[3]{-\frac{64a^9b^6c^3}{125x^{15}}},$$

$$\sqrt[4]{\frac{625a^{24}b^{12}c^4}{81x^8y^{16}}},\ \text{and}\ \sqrt[5]{\pm\frac{32}{243}p^{25}q^{20}r^{15}s^{10}t^5}.$$

4. Solve the equations

(i) $.5x+.6x+.8=.75x+.25$,

(ii) $\dfrac{x-1}{4}-\dfrac{1}{8}\left(\dfrac{x-5}{4}-\dfrac{14-2x}{5}\right)=\dfrac{x-9}{2}-\dfrac{7}{8}$,

(iii) $\left(\dfrac{a}{b}+\dfrac{b}{a}\right)x-\left(\dfrac{a}{b}-\dfrac{b}{a}\right)+2x=0.$

5. A man travelled 105 miles, and then found that if he had not travelled so fast by 2 miles an hour, he would have been 6 hours longer in performing the journey. Determine his rate of travelling.

6. Prove that

$$(a+b)^2-(c+d)^2+(a+c)^2-(b+d)^2=2(a-d)(a+b+c+d).$$

EXERCISE XV.

1. By what expression must a^2-bc be multiplied that the product may be

$$a^3+a^2b+a^2c-abc-b^2c-bc^2?$$

2. Find the square of $3a^3-5a^2b+6ab^2-2b^3$, and the cube of

$$\frac{x}{a}+\frac{a}{x}.$$

3. Extract the square root of

$$x^6-4x^5-2x^4+12x^3+9x^2.$$

4. Find the value of

$$(x+y+z)(x+y-z)(x+z-y)(y+z-x),$$

when $$z^2=x^2+y^2.$$

5. Solve the equations

(i) $\dfrac{5x-7}{2}-\dfrac{2x+7}{3}=3x-14,$

(ii) $\dfrac{2x-1}{x-2}=\dfrac{6x+4}{3x-4}.$

6. In a certain examination, three-fourths of a boy's marks were gained by translation, one-eighth by mathematics, and one-tenth by Latin prose: he also obtained one mark for French. How many marks did he obtain for each subject?

EXERCISE XVI.

1. Simplify

(i) $5a-7(b-c)-[6a-(3b+2c)+4c-\{2a-(b+2c-a)\}],$

(ii) $(2a-x)(2b-y)+(a+2x)(b+2y)-5(ab+xy).$

2. Divide

(i) $6x^4+5x^3+6x^2-17x+6$ by $6x^2-7x+2$,

(ii) $(a+b)(a+c)-(d+b)(d+c)$ by $a-d$.

3. Extract the fourth root of

$$81a^4-108a^3b+54a^2b^2-12ab^3+b^4.$$

4. Find the G.C.M. of

$$x^3-2x^2-2x-3, \text{ and } x^3+2x^2+2x+1.$$

5. Solve the equations

(i) $\dfrac{x}{6}-\dfrac{x-\frac{1}{2}}{3}-\dfrac{1}{3}\left(\dfrac{2}{5}-\dfrac{x}{3}\right)=0$,

(ii) $\dfrac{4x-8}{10}-\dfrac{20-x}{4}+\dfrac{x+\frac{1}{2}}{3}=6\frac{1}{6}$.

6. Two men receive the same sum; but if one were to receive 15 shillings more, and the other 9 shillings less, the one would receive three times as much as the other. What sum did they receive?

Exercise XVII.

1. Write down the square and cube of $-3ax^3$, and the square and cube roots of $64a^{18}b^{12}c^{-6}$. Find the value of $\sqrt{a^3+bc+2}-\sqrt{c^2-3ab+20}$, when $a=3$, $b=4$, $c=5$.

2. If $s=a+b+c$, find the value of

$$s(s-a)(s-b)(s-c),$$

and shew that if any two of the quantities a, b, c be equal, *the result is* divisible by the square of the third.

3. Prove that $\frac{a-b}{a+b}$ is contained $a^3+2a^2b+2ab^2+b^3$ times in a^3-b^3.

4. Find both the G.C.M. and the L.C.M. of

$$x^3-4ax^2+5a^2x-2a^3 \text{ and } x^3-2a^2x-4a^3.$$

5. Solve the equations

(i) $\frac{3x+1}{13}+\frac{2x-5}{3}=\frac{4x-1}{5}+\frac{2-x}{2}$,

(ii) $\frac{1}{9}\left\{3x-6-5\left(\frac{7x}{2}-5\right)\right\}+13(x-5)+\frac{1}{4}=0.$

6. A and B begin trade, A with 3 times as much stock as B. They each gain £50, and then 3 times A's stock is exactly equal to 7 times B's. What were their original stocks?

Exercise XVIII.

1. Find the numerical value of

$$\frac{a^3+b^3}{a^2-ab+b^2}+\frac{a^3-b^3}{a^2+ab+b^2},$$

where $a=\frac{1}{2}$ and $b=2$.

Simplify the expression

$$a(2b+3c)-\{c(2a+b)-b(c-2a)\}.$$

2. Divide $24x^{-4}+50x^{-3}+35x^{-2}+10x^{-1}+1$ by $x^{-1}+1$.

3. Extract the square root of

$$x^2+9+4a-6x-12\frac{a}{x}+4\frac{a^2}{x^2}.$$

4. Find the G.C.M. of

$15x^3-24x^2-9x-6$, and $5x^3+2x^2-33x+18$.

5. Solve the equations

(i) $\dfrac{x-1}{7}+\dfrac{23-x}{5}=7-\dfrac{4+x}{4}$,

(ii) $\dfrac{7x+16}{21}-\dfrac{x+8}{4x+10}=\dfrac{23}{70}+\dfrac{x}{3}$,

(iii) $\begin{cases} 5x+11y=146, \\ 11x+\ \ 5y=110. \end{cases}$

6. One-tenth of a rod is coloured red, one-twentieth orange, one-thirtieth yellow, one-fortieth green, one-fiftieth blue, one-sixtieth indigo, and the remainder, which is 302 inches long, white; what is its length?

EXERCISE XIX.

1. The product of two expressions is

$$8b^3+c^3-6abc-2a^2b-a^2c,$$

and one of them is $4b^2+c^2-2bc-ac-2ab$, find the other.

2. Find the G.C.M. of

$$x^5-x^4+x^3+2x^2-6x-4,$$
$$\text{and } x^5-x^4+x^3-2x^2-2x.$$

3. Extract the cube root of

$$8x^6+48x^5+60x^4-80x^3-90x^2+108x-27.$$

4. Resolve into elementary factors

$9a^2-25b^2$, $8a^3+27b^3$, $8a^3-27b^3$, $81a^4-256b^4$.

5. Solve the equations

(i) $\left\{\frac{1}{4}(x-2)+\frac{1}{3}\right\}-\left\{x-\frac{1}{3}(2x-1)\right\}=0,$

(ii) $x^2+9x+20=0,$

(iii) $\begin{cases} 2x+3y-4z=10, \\ 3x-4y+2z=\ \ 5, \\ 4x-2y+3z=21. \end{cases}$

6. If in a theatre $\frac{3}{8}$ of the seats are in the pit, $\frac{3}{10}$ in the lower gallery, $\frac{1}{5}$ in the upper, and there are 50 reserved seats, how many are there altogether?

Exercise XX.

1. Find the value of

$$\left(x+\frac{2x}{x-3}\right)\div\left(x-\frac{2x}{x-3}\right)$$

when $x=5\frac{1}{2}$.

2. Resolve into their elementary factors

$$x^2+9x+20,\ x^2-9x+20,\ x^2+x-20,\ x^2-x-20\ ;$$

and divide $32x^5+243$ by $2x+3$.

3. Find the G.C.M. of $x^3+14x^2y+50xy^2+7y^3$, and $x^2+6xy-7y^2$.

Also the L.C.M. of x^3-y^3, x^2-y^2, x^6-y^6, and $x-y^4$.

4. Reduce to their simplest forms

(i) $(2ab^2c^3d^{\frac{1}{6}})^3,$

(ii) $\dfrac{x^2-7x+10}{x^2-x-2},$

(iii) $\dfrac{x^3-39x+70}{x^2-3x-70}.$

5. Solve the equations

(i) $\dfrac{x-a}{b}+\dfrac{x-b}{a}=\dfrac{a^2+b^2}{ab}$,

(ii) $\begin{cases} 3x-2y=19, \\ 2x+3y=43, \end{cases}$

(iii) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=2\frac{1}{6}$.

6. 110 bushels of coals are divided among a certain number of poor persons; if each one had received one bushel more, then he would have received as many bushels as there were persons. How many persons were there?

Exercise XXI.

1. When $a=1$, $b=3$, $c=5$, $d=0$, find the value of

$$\sqrt[3]{c^2+2a}+2\sqrt[4]{c^3-5b^2+a}+\sqrt{9b^2+c^2d}.$$

Divide $\dfrac{x^2}{y^2}-\dfrac{2x}{y}-4+\dfrac{4y}{x}+\dfrac{4y^2}{x^2}$ by $\dfrac{x}{y}-\dfrac{2(x+y)}{x}$.

2. Find the square root of $x^4-2x^3+x+\dfrac{1}{4}$.

3. Find the G.C.M. of $6x^3+16x^2-12x+2$,
and $15x^3-5x^2+12x-4$,
and the L.C.M. of x^2-4, x^3+8, $2x^2+3x-2$, and $2x^2-3x-2$.

4. Simplify

(i) $\dfrac{a}{a^2-b^2}-\dfrac{1}{a+b}$,

(ii) $\dfrac{a+b+\dfrac{b^2}{a}}{a+b+\dfrac{a^2}{b}}$,

(iii) $\dfrac{x^4-x^3-2x+2}{2x^3-x-1}$.

5. Solve the equations

(i) $(x-1)(x-2)+(x-2)(x-3)=2(x-1)(x-3)-x$,

(ii) $\dfrac{x-3}{x-2}-\dfrac{x-4}{x-1}=\dfrac{7}{20}$,

(iii) $\begin{cases} 10x-2y+4z=10, \\ 3x+5y+3z=20, \\ x+3y-2z=21. \end{cases}$

6. A person sells a acres more than the m^{th} part of his estate, and there remain b acres less than the n^{th} part. Of how many acres does the estate consist?

EXERCISE XXII.

1. Multiply $3x^2-2x(x-y)-y(x+y)$ by $2x^2-3xy+y^2$, and find the continued product of

$$a+x,\ a+\frac{1}{2}y,\ a-\frac{1}{2}z:$$

deduce from the latter result the value of $(a+b)^3$.

2. Find the G.C.M. of $12x^6+30x^5+60x^4+48x^3+30x^2$ and $18x^7-9x^6+9x^5-63x^4+45x^3$.

3. Add together the fractions

(i) $\dfrac{1}{1+x},\ \dfrac{1}{1-x},\ \dfrac{2x}{x^2-1}$,

(ii) $\dfrac{a}{a-b},\ \dfrac{a}{a+b},\ \dfrac{2a^2}{a^2+b^2},\ \dfrac{4a^2b^2}{a^4-b^4}$.

4. Extract the square root of

$$x^6+4x^5+10x^4+20x^3+25x^2+24x+16.$$

5. Solve the equations

(i) $\dfrac{4x-3}{2x-1}=\dfrac{4x-7}{2x-5}$,

(ii) $\dfrac{1}{x-2}+1=\dfrac{6-x}{x^2-4}+\dfrac{1}{x+2}$.

(iii) $\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{a}, \\ \dfrac{1}{x}-\dfrac{1}{y}=\dfrac{1}{b}. \end{cases}$

6. A and B together can do a piece of work in 2 days A and C can do four times as much in 9 days; A, B, and C can do eleven times as much in 18 days. In how many days can each do it separately?

Exercise XXIII.

1. Divide

$$(b-c)a^3+(c-a)b^3+(a-b)c^3 \text{ by } c^2-ca-cb+ab.$$

2. Reduce to their simplest forms

(i) $\dfrac{(a+b)^2-(c+d)^2}{(a+d)^2-(b+c)^2}$,

(ii) $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(b-a)(b-c)}+\dfrac{1}{(c-a)(c-b)}$.

3. Find both the Greatest Common Measure, and Least Common Multiple of

(i) $3x^2-5x+2$ and $4x^3-x-4x^2+1$,

(ii) x^2+xy+y^2, $x^3+2x^2y+2xy^2+y^3$, and $x^4+x^2y^2+y^4$

4. Extract the square root of

$$x^4-2ax^3+(a^2-2b)x^2+2abx+b^2,$$

and the cube root of

$$8x^3-36x^2y+54xy^2-27y^3.$$

5. Solve the equations

(i) $\left(x-\dfrac{5}{2}\right)\left(x+\dfrac{3}{2}\right)-(x-5)(x+3)=9\dfrac{3}{4}$,

(ii) $\dfrac{2x+3}{2x+1}+\dfrac{1}{3x^2}=\dfrac{1}{x}+1$,

(iii) $\frac{1}{x^3} - 6 = \frac{1}{x}$.

6. A bag contains a certain number of sovereigns, twice as many shillings, and three times as many pence; and the whole sum is £267; find the number of sovereigns, shillings, and pence.

Exercise XXIV.

1. Resolve into their elementary factors

$x^2 + 7x + 12,\ x^2 - x - 12,\ 15x^2 - 7x - 2,\ 14x^2 - 37x + 5.$

2. Find the G.C.M. of $2x^4 - 4x^3 + 8x^2 - 12x + 6$,

and $$3x^4 - 3x^3 - 6x^2 + 9x - 3.$$

3. Simplify the expressions

(i) $\dfrac{a^2 - a - 20}{a^2 + a - 12}$, (ii) $\dfrac{\dfrac{a+p}{a-p} - \dfrac{a-p}{a+p}}{\dfrac{a+p}{a-p} + \dfrac{a-p}{a+p}}$.

4. Extract the square root of

$$16x^8 + 24x^6 + 25x^4 + 12x^2 + 4,$$

and the cube root of

$$64a^3 - 144a^2b + 108ab^2 - 27b^3.$$

5. Solve the equations

(i) $\dfrac{x-1}{4} - \dfrac{1}{8}\left(\dfrac{x-5}{4} - \dfrac{14-2x}{5}\right) = \dfrac{x-9}{2} - \dfrac{7}{8}$,

(ii) $(2x+3)(3x+2) = (3x-1)(4x-2)$,

(iii) $\begin{cases} 3x + 2y = 13, \\ 3y + 2z = 8, \\ 3z + 2x = 9. \end{cases}$

6. On a side of cricket consisting of eleven men, one-

third more were bowled than run out, and three times as many run out as stumped, two were caught out. How many were bowled, and run out, respectively?

Exercise XXV.

1. Simplify the expression

$$6a-[4b-\{4a-(6a-4b)\}].$$

Add together

$$9m-(5n+2p),\ 9n-(5p+2m),\ 9p-(5m+2n),$$

and obtain the numerical result when

$$p=2n=3m=\frac{3}{11}.$$

2. Resolve into elementary factors

$$49a^2-16b^2,\ 8x^3+27y^3,\ 16a^4-81b^4, \text{ and } (x+a)^4-(x-a)^4.$$

3. Find the G.C.M. and the L.C.M. of

$$12a^2+7ab-10b^2,\ 15a^2+2ab-8b^2 \text{ and } 15a^2+5ab-10b^2.$$

4. Simplify

$$\frac{5}{3(x-1)}-\frac{1}{6(x+2)}-\frac{3}{2x}-\frac{2x+3}{x(x-1)(x+2)}.$$

Divide $\dfrac{x^2-5x+6}{x^2-5x}$ by $\dfrac{x^2-3x}{x^2-6x+5}$.

5. Solve the equations

(i) $x+\frac{1}{5}(3x-10)=4-\frac{1}{3}(5x-10)$,

(ii) $(x+2)(x+3)=20$,

(iii) $\dfrac{5x-2}{3x+1}=\dfrac{3x+10}{4x-5}$,

(iv) $\begin{cases} 3x+4y=23, \\ 10y-2x=10. \end{cases}$

6. In a concert room 800 persons are seated on benches of equal length. If there were twenty fewer benches, it would be necessary that two persons more should sit on each bench. Find the number of benches.

Exercise XXVI.

1. Find the difference between

$$(1+x)^3+(1+x)^2y+(1+x)y^2+y^3,$$

and $$3x(x+1)+y(y+1)+2xy+1,$$

and shew by what expression this difference must be multiplied that the product may be y^4-x^4.

2 Divide $z^2+1+\frac{1}{z^2}$ by $z-1+\frac{1}{z}$.

3. Simplify the following expressions

(i) $\frac{1}{(a+b)^2}-\frac{1}{b^2-a^2}-\frac{1}{(a-b)^2}$,

(ii) $\frac{1}{x+a}+\frac{1}{x-b}+\frac{1}{x+c}$,

(iii) $\left\{\frac{a+b}{2(a-b)}-\frac{a-b}{2(a+b)}+\frac{2b^2}{a^2-b^2}\right\}\frac{a-b}{2b}$.

4. Solve the equations

(i) $\frac{1}{x-1}+\frac{2}{x-2}=\frac{3}{x-3}$,

(ii) $\frac{2}{x-1}-10x=9$,

(iii) $\frac{bx+ay}{a}=c=\frac{ax-by}{b}$.

5. A sum of £23 is divided among a certain number of persons: if each one had received 3 shillings more, he would have received as many shillings as there were persons. How many persons were there?

6. Resolve $4(xy+ab)^2-(x^2+y^2-a^2-b^2)^2$ into four factors.

EXERCISE XXVII.

1. Add together

$3x+4y+5z$, $\frac{x}{3}-\frac{y}{4}-\frac{z}{5}$, $\frac{10x}{3}-2y-3z$: and from the result subtract $x+y-\frac{z}{3}$.

2. Find the G.C.M. and L.C.M. of

(i) x^2+x-30, $x^2+11x+30$, and x^2-x-42,

(ii) $x^3+2x^2y+2xy^2+y^3$, and $x^4-x^3y-xy^3+y^4$.

3. Add together the fractions

$$\frac{1}{x+y},\quad \frac{x-y}{x^2-xy+y^2},\quad \frac{xy-x^2}{x^3+y^3},$$

and simplify the following

(i) $\dfrac{(x-y)(x^2-xy+y^2)}{x^3+y^3}\times\dfrac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$,

(ii) $\dfrac{x^6-y^6}{x^8+x^4y^4+y^8}$.

4. Extract the square root of

(i) $\dfrac{y^2}{4}-xy+3x^2-\dfrac{4x^3}{y}+\dfrac{4x^4}{y^2}$,

(ii) $x^2-4xy+11y^2-\dfrac{70xy^3-14y^4}{4x^2+4xy+y^2}$.

5. Solve the equations

(i) $\dfrac{x-4}{3}+(x-1)(x-2)=x^2-2x-4,$

(ii) $\dfrac{x-1}{x-3}+2x=12,$

(iii) $\begin{cases} 7x-2y=14+\dfrac{x}{2}, \\ 7y-2x=32+\dfrac{y}{3}. \end{cases}$

6. A company at an inn had £7. 4*s*. to pay, but before the bill was settled three of them left the room, and then those who remained had 4*s*. apiece more to pay than before; of how many did the company consist?

Exercise XXVIII.

1. Divide

$9a^2b^3-12a^4b+3b^5+2a^3b^2+4a^5-11ab^4$ by $3b^3+4a^3-2ab^2$.

2. Find the G.C.M. and L.C.M. of

$21x^2+8xy-4y^2,\ 49x^2-4y^2,\ 21x^2-20xy+4y^2,$

and $49x^2-28xy+4y^2.$

3. From $\dfrac{2x^3+1}{x-1}$ take the sum of $\dfrac{x+5}{x^2-1}$ and $\dfrac{2x^2}{x+1}$;

and multiply

$$\frac{m^2-mn+n^2}{m^3-3mn(m-n)-n^3} \text{ by } \frac{m^2-n^2}{m^3+n^3}.$$

4. Solve the equations

(i) $\dfrac{7x+9}{8}-\dfrac{3x+1}{7}=\dfrac{9x-13}{4}-\dfrac{249-9x}{14},$

(ii) $\dfrac{x-1}{x+2}+\dfrac{x-3}{x+6}=\dfrac{x-1}{x+1},$

(iii) $\begin{cases} \dfrac{2}{x}+\dfrac{3}{y}=1\dfrac{5}{12}, \\ \dfrac{3}{x}-\dfrac{4}{y}=3\dfrac{2}{3}-\dfrac{5}{x}. \end{cases}$

5. If I buy oranges at 1*s.* 6*d.* a dozen, and $3\frac{1}{2}$ times as many apples at 4*d.* a dozen; and, after mixing them, sell them at 1*s.* a dozen, and thereby gain 11*s.*; how many dozen of each do I buy?

6. Two boats start for a race; the second boat rows 25 strokes to the first's 28, but 5 strokes of the second are equal to 6 strokes of the first; if the distance between the boats = 30 strokes of the second boat, after how many strokes will it bump the first?

Exercise XXIX.

1. Multiply $x^{p(q-1)}+y^{q(p-1)}$ by $x^{p(q-1)}-y^{q(p-1)}$, and prove the result by division.

Divide $\dfrac{a^3}{125}+\dfrac{b^3}{27}$ by $\dfrac{a}{5}+\dfrac{b}{3}$.

2. Find the G.C.M. and L.C.M. of

$$12x^2-x-1, \text{ and } 6x^2-5x+1,$$

and the L.C.M. of

(i) $p^2+q^2, p^2-q^2, p^3+q^3, p^3-q^3, p^4+q^4, p^4-q^4$,

(ii) x^2-1, $7x^2+5x-2$, and $7x^2-5x-2$.

3. Extract the square root of

$$x^4-6x^3+4ax^2+9x^2-12ax+4a^2,$$

and the cube root of

$$125x^3-150x^2+60x-8.$$

4. Simplify the expressions

(i) $\dfrac{1}{2(x-1)^2}-\dfrac{1}{4(x-1)}+\dfrac{1}{4(x+1)}-\dfrac{1}{(x-1)^2(x+1)}$,

(ii) $\dfrac{\dfrac{a}{b}-\dfrac{b}{a}}{\dfrac{a}{b}+\dfrac{b}{a}-1} - \dfrac{1+\dfrac{b}{a}+\dfrac{b^2}{a^2}}{\dfrac{a}{b}-\dfrac{b^2}{a^2}}$.

5. Solve the equations

(i) $\frac{1}{3}(10x+3) - \frac{1}{2}(6x-7) = 10(x-1)$,

(ii) $\dfrac{3}{x-1} - \dfrac{x-4}{x-3} = 1$,

(iii) $\begin{cases} x+y+z=1, \\ ax+by+cz=d, \\ a^2x+b^2y+c^2z=d^2. \end{cases}$

6. An army in a defeat loses one-sixth of its number in killed and wounded, and 4000 prisoners. It is reinforced by 3000 men; but retreats, losing a fourth of its number in doing so. There remain 18,000 men. What was the original force?

EXERCISE XXX.

1. In the product of

$$1+2x+3x^2+4x^3+5x^4 \text{ by } 1+3x+5x^2+7x^3+9x^4,$$

find the coefficient of the term x^5.

2. If $a=5$, $b=4$, $c=3$, $d=2$, find the values of

$$\frac{a+b+c}{a-b+c},\ \frac{ab-cd}{ac-bd} \text{ and } \sqrt{\frac{a-1}{b-3}}.$$

3. Find the G.C.M. of

$$x^3+x^2y-3xy^2+y^3 \text{ and } x^3+3x^2y+xy^2-y^3,$$

and the L.C.M. of

$$(x+y)^2,\ x^2-y^2 \text{ and } (x-y)^3.$$

4. Simplify

(i) $\dfrac{x^2-a^2}{x^3-a^3}\left(\dfrac{x^2}{x+a}+a\right)+\dfrac{x^2-a^2}{x^3+a^3}\left(\dfrac{x^2}{x-a}-a\right)$,

(ii) $\left\{\dfrac{x-y}{x+y}+\dfrac{x+y}{x-y}\right\}\left\{\dfrac{x^2+y^2}{2xy}+1\right\}\dfrac{xy}{x^2+y^2}$.

5. Solve the following equations

(i) $\dfrac{60-x}{14}-\dfrac{3x-5}{7}=6-\dfrac{24-3x}{4}$,

(ii) $\begin{cases}\dfrac{3x+5y}{20}+\dfrac{5x-3y}{8}=3,\\[2mm] \dfrac{x+1}{y+2}=\dfrac{2}{3},\end{cases}$

(iii) $\dfrac{20}{8-x}+\dfrac{21}{6-x}=11$.

6. A boy spends his money in oranges: if he had bought 5 more for his money, each orange would have cost a halfpenny less, if 3 fewer a halfpenny more. How much did he spend?

Exercise XXXI.

1. If two numbers differ by 2, shew that the difference of their squares is twice their sum.

2. Find the G.C.M. of

$21x^3-28x^2-46x-7$, and $21x^2-58x+21$.

Find the L.C.M. of a^3-1, a^4-1, a^6-1;

and of $x^3-9x^2+23x-15$, and x^2-8x+7.

3. Simplify

$$\frac{a^3-3a^2b+3ab^2-b^3}{a^3-a^2b-ab^2+b^3},$$

and find the value of

$$\frac{x}{x^2-y^2}+\frac{y}{x^2-y^2}-\left(\frac{1}{x+y}+\frac{1}{x-y}\right) \text{ when } x=5,\ y=3.$$

4. Find the square root of

$$\frac{x^4}{9}+\frac{2x^3}{3}+\frac{4x^2}{3}+x+\frac{1}{4},$$

and the cube root of $8a^3+6a^2b+\dfrac{3ab^2}{2}+\dfrac{b^3}{8}$.

5. Solve the equations

(i) $\dfrac{1}{x-1}-\dfrac{1}{x+3}=\dfrac{1}{35}$,

(ii) $\begin{cases}3x+4y=2xy,\\4y+3z=3yz,\\6z+5x=4zx,\end{cases}$

(iii) $\begin{cases}x^2-y^2=7,\\x-y=1.\end{cases}$

6. It is between 2 and 3 o'clock; but a person looking at the clock, and mistaking the hour-hand for the minute-hand fancies that the time of day is 55 minutes earlier than the reality. What is the true time?

Exercise XXXII.

1. Shew that

$$(a+b)^2(b+c-a)(c+a-b)$$
$$+(a-b)^2(a+b+c)(a+b-c)=4abc^2.$$

2. Divide $\left(\dfrac{x^2}{a^2}+\dfrac{a^2}{x^2}-2\right)^2$ by $\dfrac{x}{a}-\dfrac{a}{x}$, and arrange the quotient in descending powers of x.

3. Simplify the following expressions

(i) $$\left(\frac{a^2+b^2}{a^2+ab+b^2}+\frac{a^2-b^2}{a^2-ab+b^2}\right)$$
$$\div\left(\frac{a^2+b^2}{a^2+ab+b^2}-\frac{a^2-b^2}{a^2-ab+b^2}\right),$$

(ii) $\frac{2}{x}-\frac{2}{x+1}-\frac{2}{(x+1)^2}+\frac{1}{(x+1)^3}$,

(iii) $\dfrac{1}{a-b+\dfrac{1}{a-\dfrac{1}{b}}}$.

4. Extract the cube root of

$$8x^3-36x^2y+48x^2z+54xy^2+96xz^2-144yz^2+108y^2z-144xyz-27y^3+64z^3.$$

5. Solve the equations

(i) $x(ax-b)(cx+d)=0$,

(ii) $\begin{cases} ay+bx=2xy, \\ cy+dx=3xy, \end{cases}$

(iii) $\begin{cases} x^2-xy+y^2=7, \\ x+y=5. \end{cases}$

6. An express train running from London to Wakefield (a distance of 180 miles) travels half as fast again as an ordinary train, and performs the distance in two hours less time; find the rates of travelling.

Exercise XXXIII.

1. Divide

$$aa'x^2+(ab'+a'b)xy+bb'y^2+(ac'^2+a'c^2)x+(bc'^2+b'c^2)y+c^2c'^2$$

by $ax+by+c^2$.

2. Find the G.C.M. of

$$x^3-2x^2-15x+36 \text{ and } 3x^2-4x-15,$$

and the L.C.M. of $6x^2-x-2$, $21x^2-17x+2$, $14x^2+5x-1$.

3. Solve the equations

(i) $x+\frac{3x-5}{4}+\frac{1-x^2}{x}=3\left\{6x-2\left(x-\frac{1}{6x}\right)\right\}$,

(ii) $\dfrac{2x+3}{3x+9}=\dfrac{2x-8}{3x-13}$.

(iii) $\begin{cases} x-y=3, \\ \dfrac{\frac{1}{y}+\frac{1}{x}}{\frac{1}{y}-\frac{1}{x}}=\dfrac{11}{3}, \end{cases}$

(iv) $\begin{cases} 3x-2y=6, \\ 3y-2z=5, \\ 3z-2x=-2. \end{cases}$

4. Find the sum, difference, and product of the roots of the equation $x^2+7x+10=0$, without solving the equation.

5. Extract the square root of

$$\frac{y^4}{4}-xy^3+3x^2y^2-4x^3y+4x^4;\text{ and of }\frac{a^2}{b^2}+\frac{b^2}{a^2}+2.$$

6. A boy swam half a mile down a stream in 10 minutes; without the aid of the stream it would have taken him a quarter of an hour. What was the rate of the stream per hour? and how long would it take him to return against it?

Exercise XXXIV.

1. If $s=\frac{1}{2}(a+b+c)$, shew that

$$(s-a)(s-b)(s-c)=s^3-\frac{s}{2}(a^2+b^2+c^2)-abc.$$

2. Find the G.C.M. of

$6x^4-2x^3+7x^2-x+2$, and $6x^4-12x^3+21x^2-6x+9$.

3. Simplify the expressions

(i) $\dfrac{1}{x^2+xy+y^2}+\dfrac{1}{x^2-xy+y^2}-\dfrac{2(x^2+y^2)}{x^4+x^2y^2+y^4}$,

(ii) $\dfrac{x^3+2x^2y+2xy^2+y^3}{x^3-y^3} \times \dfrac{x-y}{x+y}$,

(iii) $\dfrac{x^4-y^4}{x^2-2xy+y^2} \div \dfrac{x^2+xy}{x-y}$.

4. Solve the equations

(i) $\dfrac{x-\frac{1}{3}}{\frac{6}{5}} + \dfrac{\frac{x}{5}-\frac{1}{7}}{\frac{6}{7}} = 4\frac{5}{9}$,

(ii) $\dfrac{x+1}{x-1} + \dfrac{x+2}{x-2} = \dfrac{2x+13}{x+1}$,

(iii) $\dfrac{x+y}{x-y} = \dfrac{7}{3}$, and $x^2+y^2=29$.

5. Form the equations whose roots are

(i) 2, -2, and 4, (ii) 0, a, b and c.

6. A contractor undertook to build a house in 21 days, and engaged 15 men to do the work. But after 10 days he found it necessary to engage 10 men more, and then he accomplished the work one day too soon. How many days behindhand would he have been if he had not engaged the 10 additional men?

Exercise XXXV.

1. Simplify

(i) $(x+y+z)^2-(y+z)^2-(z+x)^2$
$-(x+y)^2+x^2+y^2+z^2$,

(ii) $\dfrac{\frac{1}{a}+\frac{3}{ab}}{b-1+\frac{1}{b}}$.

2. Extract the square root of

$$a^2x^2+2abxy+b^2y^2+2ac^2x+2bc^2y+c^4.$$

3. Reduce to their lowest terms

(i) $\dfrac{a^2-8ab+7b^2}{a^2-3ab-28b^2}$, (ii) $\dfrac{3x^3+x^2-5x+21}{6x^3+29x^2+26x-21}$.

4. Resolve into elementary factors

(i) $(a^2+b^2-c^2)^2-4a^2b^2$,

(ii) $m^4-n^4+2n(m^3+n^3)-(m+n)^2(m-n)^2$.

5. Solve the equations

(i) $\dfrac{x}{3}+\dfrac{3}{x}=\dfrac{x}{12}+\dfrac{12}{x}$,

(ii) $\begin{cases} x^2+xy=15, \\ xy-y^2=2, \end{cases}$

(iii) $\begin{cases} x^2yz=a, \\ xy^2z=b, \\ xyz^2=c. \end{cases}$

6. A person rents a certain number of acres of pasture-land for £70, he keeps 8 acres in his own possession, and sublets the remainder at 5*s*. an acre more than he gave, and thus covers his rent and has £2 over. How many acres were there?

Exercise XXXVI.

1. Shew that the difference of the squares of two consecutive numbers is equal to the sum of the numbers.

2. Find the L.C.M. of

(i) $10x^2-30x+20$, $15x^2-75x+90$, and $6x^2-24x+18$,

(ii) $a^4+a^2b^2+b^4$, a^3-b^3, a^3+b^3, and a^2-b^2.

3. Simplify

(i) $\dfrac{a^5-a^4b-ab^4+b^5}{a^4-a^3b-a^2b^2+ab^3}$,

(ii) $$\frac{\frac{a^2+b^2}{b}-a}{\frac{1}{b}-\frac{1}{a}} \times \frac{a^2-b^2}{a^3+b^3} \times \left(\frac{a+b}{a-b}+\frac{a-b}{a+b}\right) \times \left(\frac{a}{a+b}+\frac{b}{a-b}\right),$$

(iii) $$\frac{z}{x+y-z}-\frac{y}{z+x-y}+\frac{x}{z-y-x}.$$

4. Solve the equations

(i) $$\frac{3-x}{3+x}-\frac{2-x}{2+x}+\frac{1-x}{1+x}=1,$$

(ii) $$\frac{2x-y}{1}=\frac{2y-z}{2}=\frac{2z-t}{4}=\frac{2t-x}{8}=\frac{15}{4},$$

(iii) $$\begin{cases} \dfrac{a^2}{x^2}+\dfrac{y^2}{b^2}=6, \\ \dfrac{a}{x}\cdot\dfrac{b}{y}=1. \end{cases}$$

5. Form the equation whose roots are the reciprocals of those of the equation

$$(x+5)^2-4x=160.$$

6. What is the price of eggs per dozen, when two less in a shilling's worth raises the price one penny per dozen?

Exercise XXXVII.

1. Shew that

$$\frac{a^2b^2c^2-a^2b^2-c^2+1}{a^2bc-\frac{c}{b}+b\left(a^2-\frac{1}{b^2}\right)}+1=a^2, \text{ if } c=a+2, \text{ and } b=a-1.$$

2. Find the G.C.M. of

$$a^5+4a^4b-4a^3b^2-14a^2b^3+ab^4+15b^5,$$

and $$a^4+2a^3b-7a^2b^2+8ab^3-5b^4:$$

and the L.C.M. of

$$x^3-3x^2+3x-1, \text{ and } x^3-x^2-x+1.$$

3. Extract the square root of

$$x^2-ax-bx+\frac{a}{2}\left(\frac{a}{2}+b\right)+\frac{b^2}{4},$$

and the cube root of

$$\frac{a^3}{b^3}-\frac{b^3}{a^3}-3\left(\frac{a}{b}-\frac{b}{a}\right).$$

4. Shew that if the number 4 be divided into any two parts, their product is less than 4 by the square of half their difference.

5. Solve the equations

(i) $\left(\frac{2a}{x}-\frac{x}{a}-1\right)\left(1-\frac{a}{x}+\frac{2x}{a}\right)=0,$

(ii) $(x+3)(x+4)(x+5)=3.4.5,$

(iii) $\begin{cases} x^2+xy+y^2=7, \\ 5x+2y=9. \end{cases}$

6. Two crews row a match over a four-mile course, one pulls 42 strokes a minute, the other 38, and the latter does the distance in 25 minutes; supposing both crews to row uniformly, and 40 strokes of the former to be equivalent to 36 of the latter, find the position of the losing boat at the end of the race?

EXERCISE XXXVIII.

1. Find the G.C.M. and L.C.M. of

(i) $2x^2+xy-3y^2$, and $3x^3-x^2y-xy^2-y^3$,

(ii) $a^2+3ab+2b^2$, $a^2+ab-2b^2$, $a^3+2a^2b-ab^2-2b^3$.

2. Simplify

(i) $(2a-b)(2a+b)+[ab-b\{a-(2a-\overline{2a-b})\}]$,

3

(ii) $$\frac{a^2-ab}{a^3-b^3} \times \frac{a^2+ab+b^2}{a+b}$$
$$+\left(\frac{2a^3}{a^3+b^3}-1\right)\left(1-\frac{2ab}{a^2+ab+b^2}\right).$$

3. Extract the square root of

$$\frac{a^2}{4}+\frac{b^2}{9}+\frac{c^2}{16}-\frac{ab}{3}+\frac{ac}{4}-\frac{bc}{6},$$

and if $a^3-4ab+8c=0$, find the condition that

$$x^4+ax^3+bx^2+cx+d$$

may be a perfect square.

4. Solve the equations

(i) $x(x^2-11x+30)=0$,

(ii) $\dfrac{5x+3}{x-1}+\dfrac{2x-3}{2x-2}=9$,

(iii) $\begin{cases} 20x+2y+5z=35, \\ x+3y+7z=-31, \\ 3x+y-5z=49, \end{cases}$

(iv) $\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{5}{6}, \\ x+y=5. \end{cases}$

5. Form the equations whose roots are

(1) $2, -3, 4$, (2) $\pm 2, \pm 3$, (3) $a+b, a-b$.

6. A person buys a certain number of shares for as many pounds per share as he buys shares; after they have risen as many pence per share as he has shares, he sells and gains £15. How many shares did he buy?

Exercise XXXIX.

1. Prove that

$$\frac{a+b}{ab}\left(\frac{1}{a}-\frac{1}{b}\right)-\frac{b+c}{bc}\left(\frac{1}{c}-\frac{1}{b}\right)$$

is the difference of two squares. Find its value when

$$a=1 \text{ and } c=\frac{1}{2}.$$

2. Resolve into elementary factors

(i) $(1+a)^2(1+c^2)-(1+c)^2(1+a^2)$,

(ii) $x^4-(a+b)x^3+(a+b)abx-a^2b^2$.

3. Find both the G.C.M. and L.C.M. of

$$x^3+3x^2y+3xy^2+y^3,\quad x^3+x^2y+xy^2+y^3,$$

and

$$x^3+x^2y-xy^2-y^3,$$

and the G.C.M. only of

$$2x^5+8x^4+12x^3+20x^2+10x+12,$$

and

$$x^6-2x^5-2x^4-4x^3+x^2+6x.$$

4. Simplify

(i) $\dfrac{1}{a+x}+\dfrac{2a}{a^2+x^2}+\dfrac{4a^3}{a^4+x^4}-\dfrac{8a^7}{a^8-x^8}$,

(ii) $\dfrac{ab(x^2+y^2)+xy(a^2+b^2)}{ab(x^2-y^2)+xy(a^2-b^2)}$,

(iii) $\left(\dfrac{x^2+y^2}{x^2-y^2}-\dfrac{x^2-y^2}{x^2+y^2}\right)\div\left(\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}\right)$.

5. Solve the equations

(i) $\begin{cases}\dfrac{7x+6}{11}+y-16=\dfrac{5x-13}{2}-\dfrac{8y-x}{5}, \\ 3(3x+4)=10y-15.\end{cases}$

(ii) $\begin{cases} 3y^2 - 2x^2 = 19, \\ y^2 + xy = 15, \end{cases}$

(iii) $x^3 - 1 = 0.$

6. The sum of two numbers is 4225, and their G.C.M. is 845, shew that there are two pairs of numbers satisfying these conditions, and find them.

EXERCISE XL.

1. Simplify the expressions

(i) $\left\{ \dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}} \right.,$

(ii) $\sqrt{\{4 + \sqrt{(16x^2 + 8x^3 + x^4)}\}}.$

Determine which is the greater, $(1+x)^{2n}$, or $(1+2x)^n$.

2. If any three consecutive whole numbers be taken, prove that the sum of the squares of the greatest and least is greater by 2 than twice the square of the middle number.

3. Resolve into elementary factors

$$2xyz + x^2(y+z) + y^2(z+x) + z^2(x+y).$$

4. If α and β are the roots of the equation

$$x^2 + px + q = 0,$$

express $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}$ in terms of p and q, and form the equation whose roots are $\alpha^2 + \beta^2$, and $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$.

5. Solve the equations

(i) $\dfrac{1}{x+a+b}=\dfrac{1}{x}+\dfrac{1}{a}+\dfrac{1}{b}$,

(ii) $\begin{cases} 3x^2+2xy-y^2=180, \\ \dfrac{x+y}{3x-y}=\dfrac{5}{9}, \end{cases}$

(iii) $\begin{cases} x^2-z(2y+z)=b^2, \\ y^2-z(2x+z)=a^2, \\ a+x=b+y. \end{cases}$

6. *A* and *B* start on a walking match round a circular course at the same time from opposite points. After half an hour they are both in their original positions, *A* having gone round three times and *B* four times. Assuming that each walks with uniform speed, when and where will *B* next overtake *A*?

Exercise XLI.

1. Multiply $x^{\frac{2}{p}}-(a+b)x^{\frac{1}{p}}+ab$ by $x^{\frac{1}{p}}-c$,

and $x^{\frac{p}{2}}-x^{\frac{p}{4}}y^{\frac{q}{4}}+y^{\frac{q}{2}}$ by $x^{\frac{p}{2}}+x^{\frac{p}{4}}y^{\frac{q}{4}}+y^{\frac{q}{2}}$.

2. Extract the square root of $1+x$ to five terms, and find for what value of x the expression

$$x^4+6x^3+11x^2+3x+31$$

is a perfect square.

3. Simplify the following expressions

(i) $\dfrac{1}{a+\dfrac{1}{b+\dfrac{1}{c}}}$,

(ii) $\left\{\frac{a+b}{ab}\left(\frac{1}{a}-\frac{1}{b}\right)-\frac{b+c}{bc}\left(\frac{1}{c}-\frac{1}{b}\right)\right\}\frac{ac}{a+c}\left(\frac{1}{a}-\frac{1}{c}\right)^{-1}$,

(iii) $\left\{2\sqrt{a^{\frac{1}{3}}b^{-\frac{1}{3}}}+3\sqrt[3]{a^{\frac{1}{2}}\sqrt{b^{-1}}}-4\sqrt[3]{ab^{-1}\sqrt[3]{ab^{-1}}}\right\}^6$.

4. Solve the equations

(i) $(x+3)^3-3(x+2)^3+3(x+1)^3-x^3=x+3$,

(ii) $\begin{cases} a(x+y)+b(x-y)=2a, \\ x(a-b)-y(a+b)=-2b, \end{cases}$

(iii) $\begin{cases} xy=c(x+y+z), \\ yz=a(x+y+z), \\ xz=b(x+y+z). \end{cases}$

5. The ten's digit of a number is less by 2 than the unit's digit, and if the digits are inverted, the new number is to the former as 7 : 4. Find the digits.

6. A certain crew can pull 9 strokes to 8 of a certain other crew, but 79 strokes of the latter are equal to 90 of the former; which is the faster crew?

Exercise XLII.

1. Multiply $3\sqrt[4]{5}$ by $4\sqrt[3]{2}$, and find the continued product of $\frac{1}{2}\sqrt{3}$, $\frac{1}{3}\sqrt[3]{3}$, and $\frac{1}{4}\sqrt[6]{3}$.

Simplify

$$\sqrt{48}-\frac{6}{\sqrt{12}}-\frac{4}{3}\sqrt[4]{729}+2\sqrt[6]{27}.$$

2. Divide x^y-y^x by $x^{\frac{y}{5}}-y^{\frac{x}{5}}$; and shew that x^4+a^4 is divisible both by $x^2+\sqrt{2}ax+a^2$ and by $x^2-\sqrt{2}ax+a^2$.

3. Simplify

(i) $\frac{1}{2x^2-4x+2}+\frac{1}{2x^2+4x+2}+\frac{1}{1-x^2}$,

(ii) $\left(1+\dfrac{x}{1-x}\right)\left(1-\dfrac{x}{1+x}\right)\left(1-x^2+\dfrac{1-x^2}{x}\right)$,

(iii) $\dfrac{(x^3+y^3)\,(x^2+y^2)^2}{x^4-y^4} \div \dfrac{x^2-xy+y^2}{x+y}$.

4. Shew that there is only one value of x which makes $x^3+3cx^2+2c^2x+5c^3$ equal to the cube of $x+c$, and find it.

5. Solve the equations

(i) $\dfrac{x-2}{x+2}+\dfrac{6}{x-1}=\dfrac{x}{x-2}$,

(ii) $\sqrt{4+x}+\sqrt{x}=3$,

(iii) $\begin{cases} x+y=7, \\ x^3+y^3=91. \end{cases}$

6. The sum of two numbers multiplied by the greater is 84; their difference multiplied by the less is 10: find them.

Exercise XLIII.

1. Extract the square root of

$$x^{\frac{8}{5}}-2a^{-\frac{3}{5}}x^{\frac{11}{5}}+2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{-\frac{6}{5}}x^{\frac{14}{5}}-2a^{\frac{1}{5}}x^{\frac{7}{5}}+a^{\frac{8}{5}}.$$

2. Add together

$$\frac{2x-y}{x^2-xy+y^2},\quad \frac{2x+y}{x^2+xy+y^2},\quad \frac{4x^3+2xy^2}{x^4+x^2y^2+y^4},$$

and simplify the expression

$$\frac{\dfrac{x}{1+\dfrac{1}{x}}+1-\dfrac{1}{x+1}}{\dfrac{x}{1-\dfrac{1}{x}}-x-\dfrac{1}{x-1}}.$$

3. Find the G.C.M. of

$$np^3q + 3np^2q^2 + npq^3 - 2nq^4,$$

and $$mp^4 + mp^3q - mp^2q^2 + 2mpq^3,$$

and the L.C.M. of

$$x^3 + 3x^2 - 6x - 8,\ x^3 - 2x^2 - x + 2, \text{ and } x^2 + x - 6.$$

4. Rationalize the denominators in

(i) $3\sqrt{2\frac{1}{2}}$, (ii) $\sqrt[3]{5\frac{1}{3}}$, (iii) $\dfrac{11}{3\sqrt{2} - \sqrt{7}}$.

Extract the square root of $7 + \sqrt{13}$.

5. Solve the equations,

(i) $$x = \cfrac{4}{5 - \cfrac{4}{5 - \cfrac{4}{5 - x}}},$$

(ii) $$\sqrt{x+2} + \sqrt{3x+4} = 8,$$

(iii) $$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{6} = \frac{5}{x+y+1}.$$

6. A merchant sold 5 dozen of sherry and 8 dozen of claret for £30; he sold 2 dozen more of sherry for £12 than he sold of claret for £10: required the price of each.

Exercise XLIV.

1. Divide $x^5 - 2x^4 - 4x^3 + 13x^2 - 11x - 7$ by $x^2 - 3x + 7$, and hence prove that the former expression vanishes when $x = 1 + \sqrt{2}$, or $1 - \sqrt{2}$.

2. Resolve into two simple factors

$$2y^2 - 5xy + 2x^2 - ay - ax - a^2,$$

and find the L.C.M. of

$$3x^2-5x+2 \text{ and } 4x^3-x-4x^2+1.$$

3. If $2s=a+b+c$, prove that

(i) $(s-a)^2+(s-b)^2+(s-c)^2+s^2=a^2+b^2+c^2$,

(ii) $\dfrac{1}{s-a}+\dfrac{1}{s-b}+\dfrac{1}{s-c}-\dfrac{1}{s}=\dfrac{abc}{s(s-a)(s-b)(s-c)}$.

4. Solve the equations

(i) $2x-x^2+\sqrt{6x^2-12x+7}=0$,

(ii) $\begin{cases} \dfrac{y+3}{5}-\dfrac{x-y}{6}=2x-8, \\ \dfrac{2x-y}{7}+3x=2y-6, \end{cases}$

(iii) $\begin{cases} x+y+z=14, \\ x^2+y^2+z^2=84, \\ xz=y^2. \end{cases}$

5. An officer can form the men in his battalion into a solid square, and also into a hollow square 12 deep: if the front in the latter formation exceed the front in the former by 3, find the number of men in the battalion.

6. Replace the radical signs and negative indices by positive indices in the following expressions

(i) $\sqrt{a^2x}+\sqrt[4]{b^3x^2}+\sqrt[6]{c^4x^3}$,

(ii) $\sqrt{a\sqrt{b\sqrt{c}}}$,

(iii) $\sqrt[4]{a^2b^8}+a(\sqrt[3]{b})^6+\sqrt[5]{a^{-2}b^{10}}+\sqrt{a^{-3}b^4}$.

EXERCISE XLV.

1. Prove that $\dfrac{3a-b}{a+b+\dfrac{a-b}{1+\dfrac{a-b}{a+b}}} = \dfrac{2a}{a+b}$,

and that if $x+\sqrt{x^2-1}=y$, $2x=y+\dfrac{1}{y}$.

2. If $x^y=y^{-x}$, shew that $\left(\dfrac{x}{y}\right)^{-\frac{y}{x}}=y^{\frac{y}{x}+1}$.

Divide x^2-y^{-2} by $x^{\frac{1}{2}}+y^{-\frac{1}{2}}$.

3. Reduce to its lowest terms the fraction

$$\frac{9x^3-3x^2y-6x^2+2xy}{6x^4-3xy^2-4x^3+2y^2}.$$

4. Solve the equations

(i) $x^{\frac{3}{2}}+7(x-14x^{\frac{1}{2}})=0$,

(ii) $\sqrt{x+3}+\sqrt{x+8}=5\sqrt{x}$,

(iii) $\begin{cases} x^2y^2-18xy+72=0, \\ 6x^2-17xy+12y^2=0. \end{cases}$

5. In a race A runs at the uniform rate of 300 yards a minute, B runs over the first half of the course at the rate of 280 yards a minute, and over the last half of the course at the rate of 320 yards a minute: which wins? And what is the length of the course if the winner comes in 15 seconds before the other?

6. Simplify $\dfrac{2\sqrt{3}+4}{2\sqrt{5}-3\sqrt{2}}$, and extract the square root of $1+\sqrt{-48}$.

Exercise XLVI.

1. Prove that

$$a(a-x)(a-2x)=(a-b)(a-b-x)(a+2b-2x)$$
$$+b(b-x)(3a-2b-2x).$$

2. Extract the square root of

$$a^2+b^2+c^2+4ab+2ac+2bc-2\sqrt{2ab}(a+b+c),$$

and the cube root of

$$x^6-3ax^5+5a^3x^3-3a^5x-a^6.$$

3. If $a^{m^n}=(a^m)^n$, find m in terms of n.

Find the value of $\dfrac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, correct to 4 places of decimals.

4. Simplify

(i) $\dfrac{x^3-2x^2-5x-12}{2x^4-5x^3-11x^2-5x+4}$,

(ii) $\dfrac{\dfrac{a}{b}+\dfrac{b}{a}}{\dfrac{a}{b}-\dfrac{b}{a}} \div \dfrac{\dfrac{a^2}{b^2}-\dfrac{b^2}{a^2}}{\left(\dfrac{1}{b}+\dfrac{1}{a}\right)^2}$,

(iii) $\dfrac{5+\sqrt{7}}{4+\sqrt{7}}+\dfrac{5-\sqrt{7}}{4-\sqrt{7}}$.

5. Solve the equations

(i) $\dfrac{x^2-4}{x^2+4}+\dfrac{x^2+4}{x^2-4}=\dfrac{194}{65}$,

(ii) $\sqrt{2x+1}+\sqrt{7x-27}=\sqrt{3x+4}$,

(iii) $\begin{cases}(x+y)(x-y)=9,\\(2x+y)(2y+x)=182.\end{cases}$

6. A vessel containing water has three holes A, B, C in it: A is left open by itself 1 hour, then A and B together for 2 hours more, when the vessel is found to be empty: had B been left open by itself 2 hours, the vessel would have been emptied by B and C together in 3 hours more; and had C been left open by itself 3 hours, it would have been emptied by A and C together in 1 hour more. Find the time in which the vessel would have been emptied, if all the holes had been open together.

Exercise XLVII.

1. Find the value of $\dfrac{x^3 - x}{x^4 - x^2 + 1}$, when $2x = 1 + \sqrt{5}$.

2. Find the G.C.M. of

$$12x^6 + 30x^5 + 60x^4 + 48x^3 + 30x^2,$$

and

$$18x^7 - 9x^6 + 9x^5 - 63x^4 + 45x^3.$$

3. Simplify

(i) $\dfrac{3}{2(x-1)} - \dfrac{1}{2(x+1)} + \dfrac{x-2}{x^2+1}$,

(ii) $\dfrac{x^2}{a - \sqrt{a^2 - x^2}} - \dfrac{x^2}{a + \sqrt{a^2 - x^2}}$,

(iii) $\sqrt{14 + 6\sqrt{5}}$.

4. Solve the equations

(i) $\dfrac{2x}{3} + \dfrac{\dfrac{3x-5}{4} - \dfrac{5x-3}{6}}{\dfrac{4x-3}{9} - \dfrac{2x-5}{4}} = \dfrac{2x-4}{3}$,

(ii) $x^2 + 3x = \sqrt{x^2 + 3x} + 6$,

(iii) $\begin{cases} x^{\frac{1}{4}} + y^{\frac{1}{4}} = 6, \\ x^{\frac{3}{4}} + y^{\frac{3}{4}} = 126. \end{cases}$

5. If α, β be the roots of the equation $px^2+x+n=0$, shew that

$$\left(1+\frac{\beta}{\alpha}\right)+\left(1+\frac{\alpha}{\beta}\right)=\frac{1}{np}.$$

6. Shew that

$$\frac{5\sqrt{6}}{\sqrt{6}-1}+\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}=(3+\sqrt{3})(2+\sqrt{2}).$$

Exercise XLVIII.

1. Find the value of $a^3+b^3+3abc-c^3$

when $a=\cdot02$, $b=\cdot08$, $c=\cdot1$.

What are the values of $\left(\frac{3}{4}\right)^{m-1}$ when $m=0, 1, 2$, respectively?

Simplify $\left\{(a^m)^{m-\frac{1}{m}}\right\}^{\frac{1}{m+1}}$ and $\left(a^{1+\frac{q}{p}}\right)^{\frac{p}{p+q}}+\sqrt[p]{\frac{a^{2p}}{(a^{-1})^{-p}}}$.

2. Extract the square root of

$$x^{-4}-6x^{-3}y^{-1}+\frac{29}{3}x^{-2}y^{-2}-2x^{-1}y^{-3}+\frac{1}{9}y^{-4},$$

and the cube root of

$$x^{\frac{3}{2}}-8y^{\frac{3}{2}}z^{\frac{3}{2}}-6\sqrt{xyz}(\sqrt{x}-2\sqrt{yz}).$$

3. Simplify the expressions

(i) $$\frac{x^{\frac{1}{3}}+3y^{\frac{1}{3}}}{x^{\frac{1}{3}}-3y^{\frac{1}{3}}}+\frac{x^{\frac{2}{3}}-3x^{\frac{1}{3}}y^{\frac{1}{3}}+9y^{\frac{2}{3}}}{x^{\frac{2}{3}}+3x^{\frac{1}{3}}y^{\frac{1}{3}}+9y^{\frac{2}{3}}},$$

(ii) $$\frac{23+46\sqrt{2}}{2\sqrt{7}-\sqrt{5}},$$ (iii) $$\frac{\sqrt{1-x}+\frac{1}{\sqrt{1+x}}}{1+\frac{1}{\sqrt{1-x^2}}}.$$

4. A certain fraction becomes $\frac{1}{3}$, if 1 be added to its numerator, if 1 be added to its denominator it becomes $\frac{1}{4}$. What is the fraction?

5. Simplify $\sqrt{20} + 3\sqrt{\frac{4}{5}} + 3\left(\frac{5}{9}\right)^{-\frac{1}{2}} - \left(\frac{25}{81}\right)^{\frac{1}{4}}$ and extract the square root of $2 + \sqrt{3}$.

6. Solve the equations

(i) $x^2 + 4\cdot 8x + 2\cdot 87 = 0$, (ii) $\begin{cases} x + y = \dfrac{6}{x}, \\ x - y = \dfrac{1}{y}, \end{cases}$

(iii) $\sqrt{x} - \sqrt{a - \sqrt{ax + x^2}} = \sqrt{a}$,

(iv) $x^3 \pm 3x = a^3 \mp \dfrac{1}{a^3}$.

7. Eliminate x and y from the equations

$$ay + bx = bh, \qquad ky + hx = b^2, \qquad x^2 + y^2 = b^2.$$

Exercise XLIX.

1. Divide $x^{\frac{3}{4}} - \frac{3}{5}x^{\frac{7}{10}} - \frac{2}{3}x^{\frac{7}{12}} + \frac{2}{5}x^{\frac{8}{15}} + \frac{1}{2}x^{\frac{1}{2}} - \frac{3}{10}x^{\frac{9}{20}}$ by $\sqrt[4]{x} - \frac{3}{5}\sqrt[5]{x}$.

2. Simplify

(i) $(x-y)(x+y) - [xy - x\{y - x(y + \overline{1-y})\}]$,

(ii) $\left(1 - \dfrac{b^2}{a^2}\right)\left(1 - \dfrac{ab - b^2}{a^2}\right)\dfrac{a^4}{a^3 + b^3} \cdot \dfrac{a-b}{a^2 + b^2}$,

(iii) $\dfrac{a^{-1} + b^{-1}}{a^{-1} - b^{-1}} \times \dfrac{a^{-2} - b^{-2}}{a^{-2} + b^{-2}} \times \dfrac{1}{\left(1 + \dfrac{b}{a}\right)^{-2} + \left(1 + \dfrac{a}{b}\right)^{-2}}$.

3. Find both the G.C.M. and L.C.M. of

$x^2-3x-70$, $x^3-39x+70$, and $x^3-48x+7$.

4. Multiply $\left(\dfrac{5+2\sqrt{3}}{4-\sqrt{3}}\right)^2$ by $\left(\dfrac{2-\sqrt{3}}{\sqrt{3}+1}\right)^2$ and divide

$$3\sqrt{5}+2\sqrt{3} \text{ by } 4\sqrt{3}-3\sqrt{5}.$$

5. Solve the equations

(i) $\dfrac{b^2-ax}{b}=b-\dfrac{a^2-bx}{a}$, (ii) $\begin{cases} my+nx=axy, \\ ny+mx=bxy, \end{cases}$

(iii) $\sqrt{x+5}+\sqrt{x-3}=2\sqrt{x}$,

(iv) $x^4-2x^3=36-x^2$.

6. If α, β are the roots of the quadratic equation $ax^2+bx+c=0$, find the value of

$$\alpha^2(\alpha^2\beta^{-1}-\beta)+\beta^2(\beta^2\alpha^{-1}-\alpha).$$

7. Two trains start at the same time from two towns, and each proceeds at a uniform rate towards the other town. When they meet, it is found that one train has run 108 miles more than the other, and that if they continue to run at the same rate, they will finish the journey in 9 and 16 hours respectively. Find the distance between the towns, and the rates of the train.

Exercise L.

1. Shew that $4a^2-4b^2$ is a factor of

$$(a^2+ab-b^2)^2-(a^2-ab-b^2)^2,$$

and find the condition that $x^2+nax+a^2$ may be a factor

$$x^4+ax^3+a^2x^2+a^3x+a^4.$$

2. Find the L.C.M. of

(i) $15xy(x^2-y^2)$, $20y^2(x^2-xy+y^2)$, $30x(x^3+y^3)$, $12(x^3y-y^4)$,

(ii) $6x^2-x-2,\ 21x^2-17x+2,\ 14x^2+5x-1.$

3. Simplify

(i) $\dfrac{x^2-x-6}{x^3+4x^2+5x+2}$,

(ii) $\dfrac{\dfrac{a+b}{c+d}+\dfrac{a-b}{c-d}}{\dfrac{a+b}{c-d}+\dfrac{a-b}{c+d}}$,

(iii) $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}$.

4. Shew that

$$\sqrt{20},\ 3\sqrt{\frac{1}{5}},\ 4\sqrt{125},\ 7\left(\frac{9}{5}\right)^{\frac{1}{2}},\ 3\left(\frac{80}{9}\right)^{-\frac{1}{2}},\ \text{and}\ \sqrt[4]{\frac{25}{16}},$$

are similar surds.

Simplify $\dfrac{7(2+\sqrt{3})}{2\sqrt{3}-\sqrt{5}}$ and $\sqrt[7]{\sqrt{a^{\frac{20}{3}}b^8c^4}\sqrt[8]{a^{11}b^9c}}$.

5. Solve the equations

(i) $\begin{cases} \dfrac{x}{a}+\dfrac{y}{b}=1, \\ \dfrac{b}{x}+\dfrac{a}{y}=\dfrac{a^3}{bxy}, \end{cases}$

(ii) $10x-\dfrac{14x-9}{8x-3}=\dfrac{18-40x^2}{3-4x}-9,$

(iii) $\sqrt{x+17}+\sqrt{x-4}=\dfrac{7}{4}\sqrt{2x}.$

6. The sum of three numbers is $(p+1)(q+1)n$; the sum of the two larger is equal to p times the smallest,

and the sum of the two smaller to q times the largest. Find the numbers.

7. What is the present price of wine, when, since the price was lowered 10 shillings per dozen, we get 6 bottles more than before for £5?

EXERCISE LI.

1. Multiply $a^m - ba^{m-1}x + ca^{m-2}x^2$ by

$$a^n + ba^{n-1}x - ca^{n-2}x^2,$$

and prove your result.

2. Simplify

(i) $$\frac{\frac{b}{a^2}+\frac{1}{b}-\frac{1}{a}}{b-a} \times \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{1}{a^3}-\frac{1}{b^3}},$$

(ii) $$\frac{1}{2}\cdot\frac{\sqrt{2}+1}{\sqrt{2}-1}+\frac{1}{3}\cdot\frac{\sqrt{2}-1}{\sqrt{2}+1}.$$

3. A person has just a hours at his disposal; how far may he ride in a coach which travels b miles an hour, so as to return home in time, walking back at the rate of b' miles an hour?

4. Shew that

$$a(b+c)(b^2+c^2-a^2)+b(c+a)(c^2+a^2-b^2)$$
$$+c(a+b)(a^2+b^2-c^2)=2abc(a+b+c).$$

5. Solve the equations

(i) $$\frac{4x-7}{3}+3\tfrac{3}{4}+\frac{57-9x}{4}=241-\frac{5x-32}{4}-33x,$$

(ii) $3x^4-7x^2=43076,$

(iii) $a\left(\frac{1}{y}+\frac{1}{z}-\frac{1}{x}\right)=b\left(\frac{1}{z}+\frac{1}{x}-\frac{1}{y}\right)=c\left(\frac{1}{x}+\frac{1}{y}-\frac{1}{z}\right)$

$$=1.$$

6. A grazier bought as many sheep as cost him £60; out of these he reserved 15, and selling the remainder for £54, gained 2 shillings a head by them. How many sheep did he buy?

7. Eliminate x and y from the equations

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\quad \frac{x^2}{b^2}-\frac{y^2}{a^2}=0,\quad \frac{x^4}{a^4}-\frac{y^4}{b^4}=\frac{m}{n}.$$

Exercise LII.

1. Divide $x^{2^n}-y^{2^n}$ by $x^{2^{n-1}}-y^{2^{n-1}}$, and find the condition that x^2+ax+b^2 may be a multiple of $x+c$.

2. Find the square root of

$$x^{-4}+\frac{9x^{-2}y^{-2}}{4}+y^{-4}-x^{-3}y^{-1}-x^{-1}y^{-3},$$

and the cube root of

$$\frac{1}{27}a^{-3}-\frac{1}{6}a^{-2}b^{-1}+\frac{1}{4}a^{-1}b^{-2}-\frac{1}{8}b^{-3}.$$

3. Add together the fractions

$$\frac{1}{4a^6(a^2+x^2)},\quad \frac{1}{4a^6(a^2-x^2)},\quad \text{and}\quad \frac{1}{2a^4(a^4+x^4)}.$$

4. Prove that

$$\left(1-\frac{\sqrt{3}}{2}\right)^{-\frac{3}{2}}-\left(1+\frac{\sqrt{3}}{2}\right)^{\frac{3}{2}}=7\left(\frac{\sqrt{3}+1}{2}\right)^3,$$

and extract the square root of

$$107-42\sqrt{2}.$$

5. Form the equations whose roots are

(i) $\pm(2\pm\sqrt{3})$,

(ii) the reciprocals of the roots of the equation

$$x^4-25x^2+144=0.$$

6. Solve the equations

(i) $5x-[8x-3\{16-6x-(4-5x)\}]=6$,

(ii) $\begin{cases} x+\sqrt{x+y}=12-y, \\ x^2+y^2=41, \end{cases}$

(iii) $xyz=a^2(y+z)=b^2(z+x)=c^2(x+y)$.

7. The present income of a railway company would justify a dividend of 4 per cent., if there were no preference shares. But as £200,000 of the stock consists of such shares, which are guaranteed 5 per cent. per annum, the dividend for the ordinary shares is $3\frac{1}{2}$ per cent. What is the whole amount of stock?

Exercise LIII.

1. Shew that

$$\frac{1}{p^2+q^2}\cdot\frac{1}{x^2-p^2}-\frac{x}{(p^2+q^2)(x^2+q^2)(x+p)}$$
$$-\frac{p}{(p^2+q^2)(x^2+q^2)(x+p)}$$
$$=\frac{1}{(x^2-p^2)(x^2+q^2)}.$$

2. Find the G.C.M. of

$$x^4-9x^3+29x^2-39x+18,\quad 4x^3-27x^2+58x-39,$$
$$\text{and } x^3-8x^2+19x-12.$$

3. Simplify

(i) $\dfrac{\sqrt[11]{\{(\sqrt[3]{a^4b^2c})^5(\sqrt{a^3b^2c})^4\}^3}}{a^{\frac{6}{11}}}$,

(ii) $$\frac{\sqrt[p]{\dfrac{(xyz)^m}{\left(x^{\frac{3}{2}}y^{\frac{4}{3}}z^{\frac{5}{4}}\right)^n}}}{\left(x^{\frac{3}{2p}}y^{\frac{4}{3p}}z^{\frac{5}{4p}}\right)^{-n}}.$$

4. Simplify

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}},$$

and extract the fourth root of

$$97-56\sqrt{3}.$$

5. Solve the equations

(i) $$x=\cfrac{3}{4-\cfrac{3}{4-\cfrac{3}{4-x}}},$$

(ii) $$\frac{\sqrt{12x+1}+\sqrt{12x}}{\sqrt{12x+1}-\sqrt{12x}}=18,$$

(iii) $$\begin{cases} x^2+xy+y^2=4, \\ x^4+x^2y^2+y^4=16. \end{cases}$$

6. Find two numbers such that their sum, their product, and the difference of their squares shall all be equal.

7. In the Astronomical clock, where the hours are marked upon the dial from 1 up to 24; find the time between 8 and 9 o'clock when the hands are together.

Exercise LIV.

1. Multiply

$$\frac{1}{2}a^2\sqrt{b}-\frac{1}{3}ab\sqrt{a} \text{ by } 2a^2b+3ab\sqrt{ab},$$

and divide

$$a\sqrt{b}+b\sqrt{ac}-\sqrt{abc}-bc \text{ by } \sqrt{a}+\sqrt{bc}.$$

2. Extract the square root of

$$4x^4-4x^3+(2\sqrt{2}+1)x^2-\sqrt{2}\,x+\frac{1}{2},$$

and the cube root of

$$x^3-3x^2\sqrt{-1}-3x+\sqrt{-1}.$$

3. Simplify

(i) $$\frac{\dfrac{a^3+b^3}{a^2-b^2}}{\dfrac{a^2-ab+b^2}{a-b}},$$

(ii) $$\frac{a+b}{b}-\frac{2a}{a+b}+\frac{a^2b-a^3}{a^2b-b^3}.$$

4. Find the G.C.M. of

$$a^5+b^5+3ab(a^3+b^3)+a^2b^2(a+b),$$

$$\text{and } 11a^5-5b^5+ab(11a^3-5b^3)+6a^2b^2(a+b).$$

Find also the G.C.M. of 83772 and 367632, the values of the above expressions when $a=7$, $b=5$, and shew that it does not coincide with the numerical value of their algebraical G.C.M. What is the cause of the difference?

5. Solve the equations

(i) $$\frac{5x^2+x-3}{5x-4}=\frac{7x^2-3x-9}{7x-10},$$

(ii) $$\sqrt{x}+\sqrt{x-4}=\frac{8}{\sqrt{x-4}},$$

(iii) $$x^2-x+3\sqrt{2x^2-3x+2}=\frac{x}{2}+7.$$

6. A rectangular picture is surrounded by a narrow frame, which measures altogether 5 linear feet, and costs, at 3 shillings a foot, 10 times as many shillings as there

are square feet in the area of the picture. Find the lengt and breadth of the picture.

7. Prove that

$$\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}=\sqrt{2}.$$

Exercise LV.

1. Prove that

$$a^3(b-c)+b^3(c-a)+c^3(a-b)$$
$$+(a+b+c)(b-c)(c-a)(a-b)$$

is equal to zero.

2. Add together

$$\frac{1}{2+3x},\ \frac{2x-5}{(2+3x)^2},\ \frac{x^2-6x-7}{(2+3x)^3},$$

and from $\dfrac{1}{1+x+x^2}$ take $\dfrac{1}{1-x-x^2}$.

3. Find the L.C.M. of

(i) $x^8+x^6y^2+x^2y^6+y^8$ and $4x^5-4xy^4$,

(ii) $4x^3-4ax^2$, $3x^2-9ax+6a^2$, and $2x^3-8a^2x$.

4. Simplify

(i) $\sqrt{22-4\sqrt{10}}$,

(ii) $\dfrac{57+19\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$,

(iii) $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times(5-2\sqrt{6})$.

5. Solve the equations

(i) $7x^{\frac{1}{3}} - 20x^{\frac{1}{6}} = 3$,

(ii) $\sqrt{x^2+9} + \sqrt{x^2-9} = 4 + \sqrt{34}$,

(iii) $\begin{cases} 3x^2 - 4xy = 7, \\ 3xy - 4y^2 = 5. \end{cases}$

6. Find the integral value of x, when $\frac{4x}{3} - 1$ is not > 11, and $\frac{3x}{2} + 3$ not < 15.

7. In a time race one boat is rowed over the course at an average pace of 4 yards per second; another moves over the first half of the course at the rate of $3\frac{1}{2}$ yards per second, and over the last half at $4\frac{1}{2}$ yards per second, reaching the winning post 15 seconds later than the first. Find the time taken by each.

Exercise LVI.

1. Add together

$$\frac{a+b}{ab}(a^2+b^2-c^2),\ \frac{b+c}{bc}(b^2+c^2-a^2),\ \frac{c+a}{ca}(c^2+a^2-b^2);$$

and find the value of

$$\frac{x}{2} - \left(\frac{2x-3}{3} - \frac{3x-1}{4}\right) \div \frac{x-1}{2}$$

when $x = 4\frac{1}{3}$.

2. Simplify

$$\frac{2a^3+13a^2x-15ax^2-126x^3}{a^2x+6ax^2-7x^3} \times \frac{2a^3+19a^2x+35ax^2}{a^3-a^2x-4ax^2-6x^3}.$$

3. If two square numbers be added together, the double of the result is also the sum of two square numbers.

4. Solve the equations

(i) $\frac{x-1}{x-2} - \frac{x-2}{x-3} = \frac{x-5}{x-6} - \frac{x-6}{x-7}$,

(ii) $\begin{cases} x^3 + y^3 = 152, \\ xy(x+y) = 120, \end{cases}$

(iii) $\frac{1}{x} + \frac{1}{y} = c$, $\frac{1}{y} + \frac{1}{z} = a$, $\frac{1}{z} + \frac{1}{x} = b$,

and explain the result in (iii) when $a + c = b$.

5. Find a mean proportional to 3 and 45, and a fourth proportional to $x^2 - a^2$, $x^3 - a^3$, and $x + a$.

6. If $y^2 \propto a^2 - x^2$, and $x = 0$ when $y = b$, express y in terms of x.

7. The length of a rectangular field is to its breadth as 6 : 5; one-sixth part of the area is planted, and the remainder, 625 yards, is ploughed; what are the dimensions of the field?

EXERCISE LVII.

1. Divide $x^{-3r} - x^{-2r}y^{-p} + \frac{x^{-r}y^{-2p}}{3} - \frac{y^{-3p}}{27}$ by $x^{-r} - \frac{y^{-p}}{3}$.

2. Simplify

(i) $\frac{(ac-bd)^2 + (ad+bc)^2}{c^2+d^2} - a^2$,

(ii) $\sqrt{(9x^2 + \sqrt{36x^2 + 12x + 1})}$,

(iii) $\frac{1}{\sqrt{3} + \sqrt{2}}$.

3. Solve the equations

(i) $\frac{2x-1}{x+1} - \frac{x-7}{x-1} = 4 - \frac{3x-1}{x+2}$,

(ii) $\dfrac{(x+a)(x+a+b)}{(x+c)(x+c+b)} = \dfrac{(x-a)(x-a-b)}{(x-c)(x-c-b)}$,

(iii) $\dfrac{2x+3y-4z}{x+5} = \dfrac{3x+4y-2z}{5x} = \dfrac{4x+2y-3z}{4x-1}$

$$= \frac{x+y-z}{6}.$$

4. A person buys apples at $6d.$ a dozen, and $2\frac{1}{2}$ times as many pears at $4d.$ a dozen; after mixing them he sells them at $5d.$ a dozen, gaining $4s.$ on the whole: how many dozen of each does he buy?

5. Determine what quantity must be taken from each term of the ratio $a : b$ that it may equal the ratio $c : d$.

6. Sum the series

(i) $10+7+4+\ldots$ to 10 terms,

(ii) $\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\ldots$ to 8 terms,

(iii) $2\frac{1}{2}+1\frac{3}{4}+1+\ldots$ to 6 terms.

Insert 4 arith. means between $5\frac{1}{2}$ and $6\frac{1}{2}$.

7. A and B ride from Cambridge to Walden, a distance of 14 miles, at uniform rates; A starts 20 minutes after B, and they arrive at Walden together; coming back, they each travel one mile per hour faster than before; they start together, and although A stops a quarter of an hour at Sawston they arrive in Cambridge at the same time. Find their rates of travelling.

EXERCISE LVIII.

1. Multiply $(a^{\frac{1}{2}}+b^{\frac{3}{5}})^3$ by $a^{\frac{1}{2}}-b^{\frac{3}{5}}$, and divide $a-b$ by $\sqrt[4]{a}-\sqrt[4]{b}$.

2. Simplify

(i) $\left(\dfrac{2}{2-\sqrt{3}}\right)^{\frac{1}{2}} + \left(\dfrac{2}{2+\sqrt{3}}\right)^{\frac{1}{2}}$,

(ii) $$\frac{\dfrac{1}{1+x}}{1-\dfrac{1}{1+x}}+\frac{\dfrac{1}{1+x}+\dfrac{1}{1-x}}{\dfrac{x}{1-x}+\dfrac{x^2}{1+x}}.$$

3. Solve the equations

(i) $$\frac{x-3\frac{1}{3}}{5}+\frac{3}{x-10\frac{1}{3}}=3,$$

(ii) $$\begin{cases} x^4+x^2y^2+y^4=91, \\ x^2+xy+y^2=13, \end{cases}$$

(iii) $3x+5y=26$ in positive integers,

(iv) $\sqrt{xy}=1,\ \sqrt{yz}=24x,\ \sqrt{zx}=4y.$

4. It is between 11 and a quarter before 12 o'clock, and it is observed that the number of minute spaces between the hands is to the number 10 minutes afterwards as 3 to 2: find the time indicated by the clock.

5. Given that $z \propto x+y$, and $y \propto x^2$; and that when $x=1$, $y=2$ and $z=3$, find z in terms of x.

6. Divide 111 into three parts so that the products of each pair may be as 4 : 5 : 6.

7. The sum of an Arithmetical progression whose first term is 2 and last term 42 is 198: find the common difference and the number of terms.

Exercise LIX.

1. Prove that

$$bc(b^2-c^2)+ca(c^2-a^2)+ab(a^2-b^2)-(a+b+c)$$
$$\{a^2(b-c)+b^2(c-a)+c^2(a-b)\}=0.$$

2. Find the G.C.M. and the L.C.M. of

$$2a^6b^2+3a^5b^3-3a^4b^4-2a^3b^5+a^2b^6-ab^7$$

and $$2a^7b+3a^6b^2+a^5b^3+4a^4b^4-a^3b^5+a^2b^6.$$

3. Solve the equations

(i) $(x-15)(x+1)+(x-9)(x-7)$
$$=(x-1)(x-7)-(x-15)(23-x),$$

(ii) $\dfrac{5x-1}{\sqrt{5x}+1}=1+\dfrac{\sqrt{5x}-1}{2}$,

(iii) $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{x+y}{12}=\dfrac{7}{x+y+5}$,

(iv) $6x+5y=38$ in positive integers.

4. In how many ways can £100 be paid in crowns and guineas?

5. If $a : b :: c : d$, shew that
$$3a+4c : 5a+4c :: 3b+4d : 5b+4d,$$
and that $\quad a^4-b^4 : (a-b)^4 :: c^4-d^4 : (c-d)^4$.

6. Sum the series

(i) $1-1-3-\ldots$ to 12 terms,

(ii) $4-2+1-\ldots$ to 5 terms and to infinity.

Find the Arithmetic, Geometric, and Harmonic means between 1 and 2, and continue the series which these means form to 2 more terms.

7. A person invests in the three per cents.; after holding half a year and receiving the half-yearly dividend, he sells out, the brokerage for buying or selling being $\frac{1}{8}$ per cent. stock; the funds having fallen, he finds that the investment has been paying at the rate of $2\frac{1}{2}$ per cent. per annum. If he had waited till the end of the year when there was a further fall of $\frac{3}{8}$, he would still have received $2\frac{1}{2}$ per cent. for his investment: at what price did he buy and what fall took place?

EXERCISE LX.

1. Divide

$$6a^2-7ab+2b^2 \text{ and } 2a^3-a^2b-2ab^2+b^3 \text{ by } 2a-b:$$

also

$$(6a^2-7ab+2b^2)\,x^3+(5a^3-3a^2b-5ab^2+3b^3)\,x^2+(a^2-b^2)^2\,x$$

by $(2a-b)\,x+a^2-b^2$.

2. Extract the square root of

$$16x^6+24x^5y+9x^4y^2-16x^3y^3-12x^2y^4+4y^6,$$

and the fourth root of $24\sqrt{-1}-7$.

3. Simplify

(i) $(x+1)(x^2+x+1)^{-1}+(x-1)(x^2-x+1)^{-1}$
$$+2(x^4+x^2+1)^{-1},$$

(ii) $\left\{\dfrac{a^4-b^4}{a-b}+2ab(a+b)\right\}^{\frac{2}{3}}\div\{(a-b)^2+4ab\}^{\frac{3}{2}},$

(iii) $a.\dfrac{\sqrt{x+a}-\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}.$

4. Solve the equations

(i) $\dfrac{x+4a+b}{x+a+b}+\dfrac{4x+a+2b}{x+a-b}=5,$

(ii) $(x+1)(x+2)(x+3)=2.3.4,$

(iii) $\sqrt{3+\sqrt{x}}+\sqrt{4-\sqrt{x}}=\sqrt{7+2\sqrt{x}}.$

5. If $a:b$ be the double of the ratio of $a+c:b+c$, shew that c is a mean proportional between a and b.

6. How many terms of the series 17, 15, 13, &c., amount to 56? Shew in what particular case the answer to such a question is not double.

7. A person is walking uniformly, and when he has completed half his journey he increases his pace in the ratio of 3 to 2, and arrives at his destination 40 minutes earlier than he would otherwise have done. How long was he walking the first half?

Exercise LXI.

1. If $s = \dfrac{a+b+c}{2}$, shew that

$$a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2 = \frac{4s(s-a)(s-b)(s-c)}{b^2}.$$

2. Reduce $\dfrac{c\sqrt{ab}-ac}{bc-c\sqrt{ab}}$; and find the value of

$$\frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}}, \text{ when } x = \frac{1}{2}\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right).$$

3. Solve the equations

(i) $x(5x^2+7x-2) = x^3$,

(ii) $\begin{cases} x^2 - xy = 8x + 3, \\ xy - y^2 = 8y - 6, \end{cases}$

(iii) $x^4 + 4x^3 + 3x^2 = 2x + 2$,

(iv) $7x + 4y = 51$ in positive integers.

4. If I buy 1 goose, 2 ducks, and 3 chickens for 16*s.*, and 2 ducks and 3 chickens are together worth 3 geese, what is the price of each?

5. Shew how the principles of *Variation* are involved in the solution of the following question of Arithmetic:—

The wages of 5 men for 7 weeks being £21. 17*s.* 6*d.*, how many men can be hired to work 4 weeks for £30?

6. Sum the following series

(i) $2\frac{1}{2}+2\frac{5}{6}+3\frac{1}{6}+\ldots$ to 13 terms,

(ii) $\dfrac{n-1}{n}+\dfrac{n-2}{n}+\dfrac{n-3}{n}+\ldots$ to n terms,

(iii) $\dfrac{2}{3}-\dfrac{1}{2}+\dfrac{3}{8}-\ldots$ to 6 terms and to infinity.

What is the infinite geometric series of which the sum is $2\frac{1}{12}$ and the second term $-\dfrac{1}{2}$?

7. If a straight line be divided into two parts so that the rectangle contained by the whole line and one part is equal to six times the square of the other part; find the ratio of the parts.

Exercise LXII.

1. The product of two expressions is

$$(x+y)^3+3xy\,(1-x-y)-1,$$

and one is $x+y-1$, find without division the other, explaining the method you adopt.

2. Simplify

(i) $(\sqrt{-3x^3y})^6$,

(ii) $\sqrt{\dfrac{xy^3}{4a^2}}+\dfrac{1}{4a}\sqrt{x^3y-4x^2y^2+4xy^3}$,

(iii) $(ab^{-1}+1)\left\{a-\dfrac{1}{2}(1+\sqrt{-3})\,b\right\}$

$$\left\{a-\frac{1}{2}(1-\sqrt{-3})\,b\right\},$$

(iv) $\sqrt[4]{124-32\sqrt{15}}$.

3. Solve the equations

(i) $\dfrac{x+y}{x-y}=\dfrac{7}{3}$ and $x^2+y^2=29$,

(ii) $2\sqrt{x^2-6x+2}+4x+\dfrac{1}{5}=x^2-2x-\dfrac{4}{5}$,

(iii) $\dfrac{xy}{c}=x+y$, $\dfrac{yz}{a}=y+z$, $\dfrac{zx}{b}=z+x$.

4. If α and β are the roots of the equation

$$x^2+px+q=0,$$

shew that $\alpha^3+\beta^3=3pq-p^3$, and find the value of

$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}.$$

5. If $a : b :: c : d$, shew that

(i) $a+mb : c+md :: a-mb : c-md$,

(ii) $a : a+b :: ac-bc : ac-bd$.

6. Write down the first four terms of the series whose n^{th} term is $2+(-1)^n n$.

Find what number of terms of the series $6+9+12+\ldots$ will amount to 105, and of the series $13+10+7+\ldots$ will amount to 34.

7. In order to resist cavalry a battalion is usually formed into a hollow square, the men being four deep, but a single company is usually formed into a solid square. If the hollow of the square of a battalion, consisting of seven equal companies, is nine times as large as one of its companies' squares, find how many men there are in a company, assuming every man to occupy the same space.

Exercise LXIII.

1. Prove that $x^n-na^{n-1}x+(n-1)a^n$ is divisible by $(x-a)^2$ if n be a whole number, and x^7-a^7 by $x^2+pax+a^2$ if $p^3-p^2-2p+1=0$.

2. Find the G.C.M. of

$$(ax+by)^2-(a-b)(x+z)(ax+by)+(a-b)^2xz,$$

and

$$(ax-by)^2-(a+b)(x+z)(ax-by)+(a+b)^2xz.$$

3. Simplify

(i) $\left(\dfrac{x^2+y^2}{xy}-1\right)\left(\dfrac{x^2+y^2}{xy}+1\right)\left(\dfrac{1}{x^3+y^3}+\dfrac{1}{x^3-y^3}\right)$,

(ii) $\dfrac{1}{x-1}-\dfrac{1}{2(x+1)}-\dfrac{x+3}{2(x^2+1)}-\dfrac{4}{x^4-1}$,

(iii) $\dfrac{x-\dfrac{1}{x}}{x^{\frac{2}{3}}+x^{-\frac{2}{3}}+1}$.

4. Solve the equations

(i) $\dfrac{x^3+2x^2+1}{x^2+2x}=x+\dfrac{3}{x+2}$,

(ii) $\begin{cases} x^2(y+1)+y^2(x+1)=109, \\ xy=12, \end{cases}$

(iii) $2x+3y=24$ in positive integers.

5. If the square of x vary as the cube of y, and $x=2$ when $y=3$, find the equation between x and y.

6. If a, b, c, d be in Harmonical Progression, prove that the Harmonic mean between a and b is to that between c and d :: $3b-c : 3c-b$.

7. The two sides of a rectangle, expressed in feet, have the sum of their cubes equal to 109 times their sum, and the difference of their cubes equal to 229 times their difference: find the area of the rectangle and its diagonal.

Exercise LXIV.

1. Shew that

$$(a+b)^2+(a+c)^2+(a+d)^2+(b+c)^2+(b+d)^2+(c+d)^2$$
$$=(a+b+c+d)^2+2(a^2+b^2+c^2+d^2).$$

2. Simplify

(i) $2^{\frac{2}{3}}.6^{\frac{1}{3}}-(9)^{-\frac{2}{3}}.\sqrt[3]{3^5}-\frac{1}{2}.(576)^{\frac{1}{6}}$,

(ii) $\dfrac{\sqrt{18}}{\sqrt{3}+\sqrt{2}}+\dfrac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$,

(iii) $(9+40\sqrt{-1})^{\frac{3}{2}}\pm(9-40\sqrt{-1})^{\frac{3}{2}}$.

3. Solve the equations

(i) $$\frac{x^2+2x+2}{x+1}+\frac{x^2+8x+20}{x+4}$$
$$=\frac{x^2+4x+6}{x+2}+\frac{x^2+6x+12}{x+3},$$

(ii) $\sqrt{\dfrac{x}{4}+3}-\sqrt{\dfrac{x}{4}-3}=\sqrt{\dfrac{2x}{3}}$,

(iii) $\dfrac{2}{x}+\dfrac{1}{y}=\dfrac{3}{2},\ \dfrac{3}{z}-\dfrac{2}{y}=2,\ \dfrac{1}{x}+\dfrac{1}{z}=\dfrac{4}{3}$.

4. If x have to y the duplicate ratio of $x+z$ to $y+z$, prove that z is a mean proportional between x and y.

5. If a, b, c be in continued proportion, prove that

(i) $a : a+b :: a-b : a-c$,

(ii) $(a^2+b^2)(b^2+c^2)=(ab+bc)^2$.

6. Eliminate x and y from the equations

$$x+y=a,\ x^2+y^2=b^2,\ x^3+y^3=c^3.$$

5

7. Three men A, B, C are candidates for an office. If all had demanded a poll, the number of votes for them would have been in Arithmetical Progression, and A would have been elected by a majority equal to the number of voters for C, who has the fewest votes. C however withdraws before the election, and his supporters distribute their votes between A and B in the ratio of 1 to 4; thus A is elected by a majority of 40. Find the number of the electors.

Exercise LXV.

1. Divide

$$x^4-(a+b+c+d)\,x^3+(ab+ac+ad+bc+bd+cd)\,x^2$$
$$-(bcd+cda+dab+abc)\,x+abcd$$

by $x^2-(a+b)\,x+ab$;

also $x^{2^n}-1$ by $x^{2^{n-1}}-1$.

2. Express in their simplest forms

(i) $\dfrac{2x^2-x+2}{4x^3+3x+2}\times\dfrac{4x^2-1}{2x-1}$,

(ii) $\dfrac{2}{5-3x}+\dfrac{3x+4}{(5-3x)^2}+\dfrac{5x+6x^2+7}{(5-3x)^3}$.

3. Find a number such that when it is divided into any two parts a and b, a^2+b shall always be equal to $a+b^2$.

4. Solve the equations

(i) $\dfrac{2x+3}{3x+2}-\dfrac{2x-3}{3x-2}=1$,

(ii) $\sqrt{x+3}+\sqrt{3x-3}=10$,

(iii) $\begin{cases} x+y+z=12, \\ x+2y+3z=22, \end{cases}$ in positive integers.

5. Find a mean proportional to $2\frac{1}{2}$ and $5\frac{5}{8}$ and a third proportional to 100 and 130.

If $a : b :: c : d$, shew that

$$a : a+b :: a+c : a+b+c+d.$$

If a crew, which can row from Westminster to Battersea in 30 minutes, can row from Battersea to Westminster in 25 minutes, compare the rates of the stream and of the boat.

6. Sum the series

(i) $\frac{8}{9}+\frac{29}{63}+\frac{2}{63}+\ldots$ to 6 terms,

(ii) $\frac{2}{5}-\left(\frac{2}{5}\right)^{\frac{1}{2}}+1-\ldots$ to 5 terms and to infinity,

(iii) $1+5+13+29+\ldots$ to n terms.

7. Two boats start for a race; the second boat rows 25 strokes to 28 of the first, but 5 strokes of the second are equal to 6 of the first: if the distance between the boats is equal to 30 strokes of the second boat, after how many strokes will it bump the first?

Exercise LXVI.

1. Find the value of the continued product of

$$(1-ax)(1-bx)\{1-(a+b)x\}$$

$$\text{if } x=\frac{1}{a}+\frac{1}{a}\left(\frac{1}{\frac{a+b}{a}}-\frac{1}{\frac{a+b}{b}}\right).$$

2. Solve the equations

(i) $$\begin{cases}\dfrac{3x-2y}{3}=6-x,\\[2mm] \dfrac{8x-6y}{y}=\dfrac{14}{3},\end{cases}$$

(ii) $\dfrac{4x^3+1}{x+4} = \dfrac{2x^2}{x+1}$,

(iii) $\begin{cases} x^2 - 6\sqrt{x^2y} = 27, \\ x - 2\sqrt{xy} = 3. \end{cases}$

3. Rationalize $\dfrac{\sqrt{10}}{\sqrt{2} - \sqrt[3]{3}}$, and extract the square root of $10\frac{1}{2} - 2\sqrt{5}$.

4. The amount of glass in a window, the panes of which are in every respect equal, varies as the number, length and breadth of the panes jointly. Shew that if the whole area of glass varies as the length of the panes directly, their breadth will vary as their number inversely.

5. If $a : b :: c : d$, shew that

(i) $a^2 + 3ab + b^2 : c^2 + 3cd + d^2 :: 2ab + 3b^2 : 2cd + 3d^2$,

(ii) $a(a+b+c+d) = (a+b)(a+c)$.

6. If a and b are respectively the arithmetic and geometric means between two numbers, find the numbers in terms of a and b.

Find two harmonic means between 3 and 4.

7. A man invested £2000 in cottages: a certain number were then burnt down, but, in consequence of their having been insured, the loss on each cottage was only 10 per cent. on its cost price, and was also found to be at the rate of £10 on each cottage bought: some time after the remainder were sold 20 per cent. dearer than they had been bought, and the total gain was £100. How many cottages were originally bought, and how many were burnt?

Exercise LXVII.

1. Resolve $x^2 - y^2 - z^2 + 2yz + x + y - z$ into its two *component real* factors each of the first degree in x, y, z.

2. Simplify

(i) $\left(1-\frac{2ab}{a^2+b^2}\right)\div\left(\frac{a^3-b^3}{a-b}-3ab\right)$,

(ii) $\frac{a+\sqrt{a^2-1}}{a-\sqrt{a^2-1}}-\frac{a-\sqrt{a^2-1}}{a+\sqrt{a^2-1}}$,

(iii) $\sqrt{\frac{a^2}{b^2}+\frac{b^2}{4a^2}-\frac{b}{a}+\frac{2a}{b}}$.

3. Solve the equations

(i) $$\frac{x-4\frac{2}{3}}{3}-\frac{2x-3\frac{2}{3}}{4}=\frac{3}{2}\left(x-\frac{x-1\frac{1}{2}}{2}\right)$$
$$+\frac{4x}{3}\left\{x-3-\frac{(x-1)(x-2)}{x}\right\},$$

(ii) $\sqrt{2x+7}+\sqrt{3x-18}=\sqrt{7x+1}$,

(iii) $\frac{6y-4x}{3z-7}=\frac{5z-x}{2y-3z}=\frac{y-2z}{3y-2x}=1$,

(iv) $5x+8y=63$, in positive integers.

4. Obtain the equation whose roots are

$$\frac{\sqrt{m}}{\sqrt{m}+\sqrt{m-n}} \text{ and } \frac{\sqrt{m}}{\sqrt{m}-\sqrt{m-n}}.$$

5. A person travelling in a railway train, at the rate of 20 miles an hour, meets another train which passes him in 3 seconds; had it been moving in the other direction it would have passed him in 27 seconds: find its length.

6. If $a+b \propto a-b$, prove that $a^2+b^2 \propto ab$.

7. The sum of three quantities in geometrical progression is $\frac{13}{6}$, and the sum of their reciprocals $\frac{26}{3}$: find the quantities.

Exercise LXVIII.

1. Multiply

$$\frac{a^{3r}}{8}-\frac{a^{2r}b^{-m}}{4}+\frac{a^{r}b^{-2m}}{2}-b^{-3m} \text{ by } \frac{a^{r}}{2}+b^{-m},$$

and prove the result by division.

2. Reduce the fractions

$$\frac{2x^4+x^3+x^2+3x-2}{6x^4-7x^3+4x^2-3x+1} \text{ and } \frac{x+\sqrt[3]{xy^2}-\sqrt[3]{x^2y}}{x+y}$$

to their lowest terms.

3. Find the value of

$$\left(\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\right)^{\frac{1}{2}}$$

correct to 4 places of decimals, and simplify

$$\sqrt{12+6\sqrt{3}}+\sqrt{9-6\sqrt{2}}.$$

4. Solve the equations

(i) $\dfrac{4x-17}{x-4}+\dfrac{10x-13}{2x-3}=\dfrac{8x-30}{2x-7}+\dfrac{5x-4}{x-1}$,

(ii) $(2x+a)^3-(x+a)^3=4a(7x^2+9ax+3a^2)$,

(iii) $\begin{cases} y(x^2+y^2)=a(x+y)^2, \\ xy=a(x+y). \end{cases}$

5. Eliminate x, y, z from the equations

$$y^2+z^2=2ayz,\ z^2+x^2=2bzx,\ x^2+y^2=2cxy.$$

6. If b be a mean proportional between a and c, prove that

(i) $3a+7b : 3b+7c :: 5a-7b : 5b-7c$,

(ii) $a+2b+c : 1 :: (b+c)^2 : c$.

7. Sum the series

(i) $6-2-10-\dots$ to 7 terms,

(ii) $\left(\frac{1}{2}\right)^{-\frac{1}{2}}+1+\left(\frac{1}{2}\right)^{\frac{1}{2}}+\dots$ to infinity.

The harmonic mean between two quantities is $\frac{4}{5}$ of the geometric, and the arithmetic mean is 5: find the quantities.

Exercise LXIX.

1. Simplify

$$a^2+b^2+c^2-(a-b+c)(a+b-c)-(b-c+a)(b+c-a)$$
$$-(c-a+b)(c+a-b).$$

2. Find the G.C.M. and L.C.M. of

$$3x^5-3x^4-53x^3-43x^2+34x+30$$

and
$$3x^5+3x^4-53x^3+43x^2+34x-30.$$

3. Solve the equations

(i) $\frac{x+y}{3}=\frac{y+z}{5}=\frac{z+u}{8}=\frac{u-x}{4}$

and $3z+u-2x-4y=8$,

(ii) $$\begin{cases} \sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1, \\ \frac{x}{a}+\frac{y}{b}=1, \end{cases}$$

(iii) $\frac{a-\sqrt{2ax-x^2}}{a+\sqrt{2ax-x^2}}=b.$

4. A person has £12. 4*s*. in half-crowns, florins, and shillings, and the number of coins of each kind are respectively as the numbers 7, 5, and 3. Find the number of coins of each kind.

5. If $\dfrac{a_1}{b_1}=\dfrac{a_2}{b_2}=\dfrac{a_3}{b_3}$, shew that

$$\frac{a_1a_2+a_2a_3+a_3a_1}{b_1b_2+b_2b_3+b_3b_1}=\frac{a_1^2}{b_1^2},$$

and that

$$\frac{\sqrt{ma_1}+\sqrt{nb_1}}{\sqrt{ma_1}-\sqrt{nb_1}}=\frac{\sqrt{ma_2}+\sqrt{nb_2}}{\sqrt{ma_2}-\sqrt{nb_2}}=\frac{\sqrt{ma_3}+\sqrt{nb_3}}{\sqrt{ma_3}-\sqrt{nb_3}}.$$

6. Given that y is equal to the sum of two quantities, one of which varies as x, and the other as $\dfrac{1}{x}$, and that when $x=a, y=p$, and when $x=b, y=q$; express y in terms of x.

7. Of five numbers the first three are in Arithmetic Progression, and the first, third and fourth are in Arithmetic Progression also; the second, fourth and fifth in Geometric Progression; the last three in Harmonic Progression; and the sum of the third and fifth is 16; find the numbers.

Exercise LXX.

1. Extract the square root of

$$\frac{y^2}{x}+\frac{x^2}{4y}+\frac{2y^{\frac{3}{2}}-x^{\frac{3}{2}}}{(xy)^{\frac{1}{4}}},$$

and find the value of

$$\frac{\sqrt{az^2-a^2}}{z},\ \text{when}\ z=\frac{\sqrt{ay^2-a^2}}{y}\ \text{and}\ y=\frac{\sqrt{ax^2-a^2}}{x}.$$

2. Reduce to their simplest forms

$$\text{(i)}\quad \left(\frac{1+x}{1-x}+\frac{4x}{1+x^2}+\frac{8x}{1-x^4}-\frac{1-x}{1+x}\right)$$

$$\div\left(\frac{1+x^2}{1-x^2}+\frac{4x^2}{1+x^4}-\frac{1-x^2}{1+x^2}\right),$$

(ii) $(\sqrt{x}+\sqrt{y}+\sqrt{z})(-\sqrt{x}+\sqrt{y}+\sqrt{z})$
$(\sqrt{x}-\sqrt{y}+\sqrt{z})(\sqrt{x}+\sqrt{y}-\sqrt{z})$.

3. Solve the equations

(i) $x^2+x^{-2}=7$,

(ii) $\begin{cases} x+y-8=0, \\ \dfrac{x-y}{2}+\dfrac{2x-y}{3}+\dfrac{4}{3}=0, \end{cases}$

(iii) $ax+yz=ay+zx=az+xy=c^2$.

4. If a^2, b^2, c^2 be in arithmetical progression, then will $b+c$, $c+a$, $a+b$ be in harmonical progression.

5. Shew that

$$(ay-bx)^2+(cx-az)^2+(bz-cy)^2$$
$$=(a^2+b^2+c^2)(x^2+y^2+z^2)-(ax+by+cz)^2.$$

6. Find the number of ways in which the sum of £5 can be paid in exactly 50 coins, consisting of half-crowns, florins, and fourpenny pieces.

7. A constable is in pursuit of a thief, and proceeding at an uniform pace he finds by enquiry that the thief is travelling $1\frac{1}{2}$ miles per hour quicker than himself; he therefore doubles his speed after the first 4 hours, and takes the thief in 6 hours and 20 minutes from the commencement of the pursuit. Given that the thief had an hour's start, and never varied his speed, find the rates of travelling of the two persons, and the distance at which the capture took place.

Exercise LXXI.

1. Express

$$(a-c)x^2-(b-a)y^2+(c-b)z^2-axy-byz-czx$$

in three terms of which a, b, c are respectively factors.

In the continued product $(x-2)(2x-3)(3x-4)(4x-5)$ find the coefficient of x^3.

2. Find the G.C.M. of

$$x^3+(5m-3)x^2+(6m^2-15m)x-18m^2$$

and $$x^3+(m-3)x^2-(2m^2+3m)x+6m^2.$$

3. Find the square root of

$$\frac{5}{6}+\sqrt{\frac{2}{3}},$$

and rationalize the denominator of the fraction

$$\frac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}}.$$

4. Solve the equations

(i) $$\frac{7x-1}{5}-\left\{\frac{3x+4}{7}-\left(\frac{9x-12}{15}-\frac{30-3x}{6}\right)\right\}=\frac{5x}{4},$$

(ii) $$\begin{cases}\sqrt{x}+\sqrt{y}=14,\\ xy=576,\end{cases}$$

(iii) $$\begin{cases}x+\dfrac{y}{2}+\dfrac{z}{3}=17,\\ x+\dfrac{y}{7}-\dfrac{z-2}{5}=6,\\ \dfrac{x}{3}-y+\dfrac{z+13}{2}=\dfrac{1}{2}.\end{cases}$$

5. Find a number the sum of whose digits is 9, and whose digits, if reversed, form a number $\frac{1}{5}$ greater than the original number.

6. Given that the volume of a sphere varies as the *cube of its radius*, prove that the volume of a sphere whose

radius is 6 inches is equal to the sum of the volumes of three spheres whose radii are 3, 4, 5 inches respectively.

7. Find an Arithmetic, Geometric, and Harmonic mean between 2 and 8, and continue to 2 terms the series which these means form.

8. If the number of Permutations that can be formed with a certain number of things taking 3 together is equal to 20 times the number, how many things are there?

Exercise LXXII.

1. Multiply

$$\frac{3}{2}(x^{\frac{3}{2}} + x^{-\frac{3}{2}}) + \frac{1}{2}(x^{\frac{1}{2}} + x^{-\frac{1}{2}}) \text{ by } \frac{2}{3}(x + x^{-1}) + 1,$$

and simplify

$$\{(x+1)\,y-(x-1)\}^2-\{(y+1)\,x-(y-1)\}^2,$$

bracketing the coefficients of the different powers of x in the result.

2. Find the G.C.M. of

$$3x^{\frac{3}{2}}-5x-12x^{\frac{1}{2}}+20 \text{ and } 4x^{\frac{3}{2}}-7x-16x^{\frac{1}{2}}+28,$$

and the L.C.M. of

$$4x^3-4ax^2,\ 3x^2-9ax+6a^2, \text{ and } 2x^3-8a^2x.$$

3. Solve the equations

$$\text{(i)}\quad \frac{(1-x)(1-2x)}{2}+\frac{(1+x)(1+3x)}{3}=\frac{(1+2x)(3x-1)}{6}+\frac{(1+x)(4x-1)}{4},$$

$$\text{(ii)}\quad \begin{cases} 2x^2+3xy=26, \\ 3y^2+2xy=39\,; \end{cases}$$

and find the least positive integral solution of

$$13x-7y=31.$$

4. *A* can do half as much work as *B*, *B* half as much as *C*, and together they can complete a piece of work in 66 days, in what time could they singly do the work?

5. Shew that

$$\frac{bc}{a}+\frac{ac}{b}+\frac{ab}{c}$$

is $> a+b+c$.

6. If $$\frac{a}{b}=\frac{c}{d}=\frac{e}{f},$$

shew that $$\left(\frac{ma^n+nc^n+pe^n}{mb^n+nd^n+pf^n}\right)^{\frac{1}{n}}=\frac{a}{b},$$

and if $$\frac{c}{d} \text{ be } < \frac{a}{b} \text{ but } > \frac{e}{f},$$

then $$\frac{a+c+e}{b+d+f} \text{ lies between } \frac{a}{b} \text{ and } \frac{e}{f}.$$

7. Sum the series

$$\left.\begin{array}{l}\text{(i) } 2+5+8+\ldots \\ \text{(ii) } \frac{5}{2}+2+\frac{3}{2}+\ldots\end{array}\right\} \text{to 14 terms,}$$

(iii) $9+3+1+\ldots$ to five terms and to infinity. Insert 4 harmonic means between 2 and 12.

If l, m, n be respectively the second, fourth, and sixth terms of a G.P. shew that

$$\frac{l}{m}=\frac{m}{n}.$$

8. If eggs are 15 for a shilling, how many different shillings' worth can be picked out of a basket of 48? In how many of these will a particular egg be found?

Exercise LXXIII.

1. Simplify the following expressions:

(i) $(x^m)^{\frac{1}{m}} - (x^{1+\frac{1}{m}})^{\frac{m}{m+1}} + \sqrt[m]{x^{2m}}$,

(ii) $x^2 - [x^2 + y^2 + (x-y)(x+y) - \{x^2 - (y^2 - \overline{x^2 + y^2})\}]$,

(iii) $$\left(\frac{x^2}{y^2} - 1\right)\left(\frac{x}{x-y} - 1\right) + \left(\frac{x^3}{y^3} - 1\right)\left(\frac{x^2 + xy}{x^2 + xy + y^2} - 1\right).$$

2. Solve the equations

(i) $$x + \cfrac{1}{x + a + \cfrac{1}{x+b}} = x + \cfrac{1}{x - a + \cfrac{1}{x-b}},$$

(ii) $$\begin{cases} x + y = 6, \\ (x^2 + y^2)(x^3 + y^3) = 1440. \end{cases}$$

3. Find two numbers in the proportion of 9 to 7 such that the square of their sum shall be equal to the cube of their difference.

4. Simplify

(i) $$\frac{7(2 + \sqrt{3})}{2\sqrt{3} - \sqrt{5}},$$

(ii) $$\frac{(\sqrt{2} + \sqrt{3})(\sqrt{3} + \sqrt{5})(\sqrt{5} + \sqrt{2})}{(\sqrt{2} + \sqrt{3} + \sqrt{5})^2}.$$

5. If $a : b :: c : d$, shew that

$$\sqrt{a-b} : \sqrt{c-d} :: \sqrt{a} - \sqrt{b} : \sqrt{c} - \sqrt{d},$$

and if d be the least term in the proportion, that

$$\frac{1}{a} + \frac{1}{d} > \frac{1}{b} + \frac{1}{c}.$$

6. The sum of n terms of an Arithmetic series is

$$n\,(b^2+x^2)-n\,(n-3)\,bx,$$

find the r^{th} term and determine the series.

7. How many permutations can be made of the letters in the word "Alellein" taken all together? How many of those in the word "Ilicifolia"?

8. Expand $(3a-2b)^5$, and $\dfrac{1}{\sqrt[3]{1-x^2}}$ to 5 terms.

Exercise LXXIV.

1. If $z=1+\dfrac{b-a}{x+a}$, simplify the expression

$$\frac{(x+a)^{-2}\,(z-1)^{m+n-2}}{z^n\,(b-a)^{m+n-2}}.$$

2. Reduce to its lowest terms

$$\frac{7x^3-2x^2y-63xy^2+18y^3}{5x^4-3x^3y-43x^2y^2+27xy^3-18y^4}.$$

3. Solve the equations

(i) $\dfrac{a+b}{x+b}+\dfrac{a+c}{x+c}=\dfrac{2\,(a+b+c)}{x+b+c}$,

(ii) $\sqrt{x}+\sqrt{4+x}=\dfrac{2}{\sqrt{x}}$,

(iii) $\begin{cases} x+y+z=12, \\ xy+yz+zx=47, \\ x^2+y^2-z^2=0. \end{cases}$

4. In how many different ways is it possible to pay £100 in Napoleons (worth 16*s.* each), and five-franc pieces (worth 4*s.* each)?

5. If $ax^2+by \propto cx^2-dy$, shew that $x \propto \sqrt{y}$; a, b, c, d being constants.

6. Sum the series

(i) $5+10+15+\ldots$ to 11 terms,

(ii) $2+\frac{1}{2}-1-\ldots$ to 8 terms,

(iii) $5+1+\frac{1}{5}+\ldots$ to 5 terms and to infinity.

The first two terms of a series in harmonical progression are 4 and 3, continue the series to 3 more terms.

7. Find the coefficient of x^7 in the expansion of $\left(1-\frac{x}{2}\right)^{15}$, and expand $(2+3x)^{-\frac{2}{3}}$ to four terms.

8. A certain crew, who row 40 strokes per minute, start at a distance equivalent to five of their own strokes behind another crew, who row 45 strokes per minute. Assuming that 5 strokes of the hindmost crew are equivalent to 6 of the foremost, in how long will the former overtake the latter?

Exercise LXXV.

1. Find the value of

$$\frac{1-x}{1+x}+\frac{(1-x)(1-x^2)}{(1+x)(1+x^2)}+\frac{(1-x)(1-x^2)(1-x^3)}{(1+x)(1+x^2)(1+x^3)}.$$

2. Solve the equations

(i) $(x^2+1)(x+2)=2$,

(ii) $\begin{cases} (x+5)(y+7)=(x+1)(y-9)+112, \\ 2x+10 \quad = 3y+1, \end{cases}$

(iii) $\begin{cases} \dfrac{x}{y}-\dfrac{y}{x}=\dfrac{24}{35}, \\ x-y=2. \end{cases}$

3. Simplify

(i) $\dfrac{(2+\sqrt{3})^{\frac{3}{2}}+(2-\sqrt{3})^{\frac{3}{2}}}{\sqrt{6}}$,

(ii) $\dfrac{x+a+\sqrt{x^2-a^2}}{x+a-\sqrt{x^2-a^2}}$.

4. If x is to y in the duplicate ratio of a to b, and a to b in the subduplicate ratio of $a+x$ to $a-y$, shew that

$$x+y : x-y :: x+a : x.$$

5. If in an infinite Geometric series any term be equal to the sum of all that succeed it, shew that the common ratio will be $\frac{1}{2}$.

6. A boat club consists of 12 members, how many different crews of 8 can be formed, and from how many of these must an individual member be excluded?

7. Expand $\left(\dfrac{x}{2}-2y\right)^4$; write down the 8^{th} term of $(5x-4y)^{12}$, and the n^{th} term of $(1-x)^{-\frac{1}{2}}$.

8. Transform 5279 from the denary scale to the senary. The number 136 in the denary scale is expressed by 253 in another scale, what is the radix of this scale?

EXERCISE LXXVI.

1. Multiply

$$x^{\frac{r}{2}} + 2x^{\frac{r}{4}}y^p + 3y^{2p} \text{ by } x^{\frac{r}{2}} - 2x^{\frac{r}{4}}y^p + 3y^{2p};$$

and prove the result by division.

2. Extract the square root of

$$\left(xy\right)^{\frac{1}{2}}\left(x^{\frac{1}{2}} + 4y^{\frac{1}{12}}\right) + \frac{4y^{\frac{1}{3}}}{x^{\frac{1}{2}}} + \frac{1}{x} + 2y^{\frac{1}{4}}\left(1 + 2y^{\frac{5}{12}}\right),$$

and the cube root of

$$x^{\frac{3}{5}} - a^{\frac{1}{10}}x^{\frac{2}{5}} + \frac{a^{\frac{1}{5}}x^{\frac{1}{5}}}{3} - \frac{a^{\frac{3}{10}}}{27}.$$

3. Solve the equations

(i) $\dfrac{3x+1}{3(x-5)} - \dfrac{2x-7}{2x-8} - \dfrac{5}{2} = 0,$

(ii) $\begin{cases} x - y = a, \\ y^2 + ay + bx = 0, \end{cases}$

(iii) $\dfrac{3x + \sqrt{4x - x^2}}{3x - \sqrt{4x - x^2}} = 2,$

(iv) $3x + 5y = 17$ in positive integers.

4. Two men, A and B, sell a quantity of wheat for £28. 8*s.*: B sells 4 quarters more than A, and if he had sold the quantity that A sold would have received £10 for it, while A would have got 16 guineas for what B sold. Find the quantity of wheat sold by each, and the rate at which they sold it. Interpret all the solutions.

5. If $a : b :: c : d$, shew that

$$\frac{2a+3b}{3a+4b} = \frac{2c+3d}{3c+4d};$$

and find x if $4 + x : 2x + 8 :: 2x - 1 : 3x + 2.$

6. The sum of an infinite geometric series is $2\frac{1}{12}$, and the second term $-\frac{1}{2}$, find the series.

Sum the series

(i) $-9-7-5-\ldots$ to 10 terms,

(ii) $2-\frac{1}{2}+\frac{1}{8}-\ldots$ to 6 terms and to infinity,

(iii) $1+7+25+79+\ldots$ to 6 terms.

7. In the *Adelphi* of Terence there are twelve characters, six of which are to be undertaken by six boys: how many different arrangements can be made? In how many of these will the same boy undertake one particular character?

8. Express 47 in the scale of 5; then square the transformed number; extract the square root of the square, and reconvert the number to the scale of 10.

Exercise LXXVII.

1. Find the L.C.M. of x^3+3x^2-6x-8, x^3-2x^2-x+2, and x^2+x-6.

If the L.C.M. of a and b = the G.C.M. of c and d, shew that

$$\frac{\text{G.C.M. of } a \text{ and } b}{\text{L.C.M. of } c \text{ and } d}=\frac{ab}{cd}.$$

2. Solve the equations

(i) $ax+2\sqrt{x^2-ax+a^2}=x^2+2a$,

(ii) $x^2+a\sqrt{x^2-b^2}=x\{a+\sqrt{x^2-b^2}\}$,

(iii) $\dfrac{x+y}{xy}=\dfrac{5}{6}$, $\dfrac{x+z}{xz}=\dfrac{3}{4}$, $\dfrac{y+z}{yz}=\dfrac{7}{12}$.

3. Which is the greater ratio

$$1+x:1-x \text{ or } 1+x^2:1-x^2,$$

x being less than 1, but positive?

Three men, whose capacities for work are as the numbers 3, 4, 5, can complete a piece of work in 60 days, in what time could they singly do the work?

4. What must be the relation between a, b, and c that they may be respectively the p^{th}, q^{th}, and r^{th} terms of a geometric series?

5. At an election where every voter may vote for any number of candidates not greater than the number to be elected, there are 6 candidates, and 4 members to be chosen; in how many ways may a man vote?

6. Expand $(x+y)^6$, and $(3x-2y)^{-4}$ to 5 terms. Write down the fourth term of $(3a+2b)^{10}$ and of $(1+x)^{\frac{1}{2}}$.

7 A certain number is equal to 125 in the scale whose radix is x, to 78 in the scale whose radix is y, and to 49 in that whose radix is $x+y$; find the number.

8. A and B, starting from the same angle, run a race in opposite directions round a rectangular field; the prize being as many shillings as yards by which the winner can get beyond the starting point before meeting the other the second time. A allows B a start of 21 yards, and his speed exceeds B's by $\frac{1}{10}$th: he is 3 yards short of the opposite angle when he first meets B. Which wins? how many shillings does he gain? and how far does he run?

What is the greatest possible area of the field?

Exercise LXXVIII.

1. Shew that the coefficient of x in the expansion of $(x-a)(x-b)(x-c)$ is the coefficient of x^2 in the expansion of

$$\left(x+\frac{1}{a}\right)\left(x+\frac{1}{b}\right)\left(x+\frac{1}{c}\right)a\,.\,b\,.\,c.$$

2. Simplify

(i) $\dfrac{3^{\frac{1}{2}}}{2^{-3}}-5\sqrt{12}+18(3)^{-\frac{3}{2}},$

(ii) $\dfrac{5+\sqrt{7}}{4+\sqrt{7}}+\dfrac{5-\sqrt{7}}{4-\sqrt{7}}$.

3. Solve the equations

(i) $$\begin{cases} x+y=\dfrac{9}{x-y}, \\ x^2+y^2=\dfrac{820}{xy}, \end{cases}$$

(ii) $\sqrt{9+x^2}+\sqrt{9-x^2}=a$.

4. Find the conditions that the equation $ax^2+bx+c=0$ may have (i) roots equal in magnitude and of opposite sign, (ii) one root positive and the other negative, (iii) both roots positive, (iv) both roots negative.

What are the natures of the roots of the equations

(i) $x^2+5x+6=0$, (ii) $2x^2-9=0$?

5. Find a number greater than 30 and less than 40, such that if 9 be added to it the digits will be inverted. Are there any other numbers whose digits are inverted by the addition of 9?

6. If $$\frac{x_1}{x_2}=\frac{x_2}{x_3}=\frac{x_3}{x_4}=\dots\dots=\frac{x_n}{x_{n+1}},$$

prove that $$\left(\frac{x_1+x_2+x_3+\dots\dots+x_n}{x_2+x_3+x_4+\dots\dots+x_{n+1}}\right)^n=\frac{x_1}{x_{n+1}}.$$

7. Shew that the number of combinations of n things taken p together + the number of combinations of n things taken $p-1$ together = the number of combinations of $n+1$ things taken p together.

$$\frac{\text{The No. of comb. of } n+2 \text{ things 4 together}}{\text{The No. of comb. of } n \text{ things 2 together}}=11, \text{ find } n.$$

8. Expand

$$(1-3x)^5, \quad \frac{1}{(1-3x)^3}, \quad \frac{1}{\sqrt[3]{1-3x}}, \text{ the last two to 4 terms.}$$

Approximate to the square root of 99 by the Binomial Theorem (3 places of decimals).

EXERCISE LXXIX.

1. If $c^2=a^2+b^2$, shew that

$$(a+b+c)(a+b-c)(a+c-b)(b+c-a)=4a^2b^2,$$

and if $\quad e^x=y+\sqrt{1+y^2}$, that $2y=e^x-e^{-x}$.

2. Extract the square root of

$$\frac{1}{25}\frac{a^4}{b^2c^2}+\frac{3}{25}\frac{a^3}{bc}+\frac{13}{100}a^2+\frac{3}{50}abc+\frac{b^2c^2}{100},$$

and the cube root of

$$8a^6-36a^5+66a^4-63a^3+33a^2-9a+1.$$

3. Solve the equations

(i) $$\frac{a}{a-2x}-\frac{12x}{a-3x}=\frac{16x}{4x-a}+\frac{a}{a-6x},$$

(ii) $$\frac{\sqrt{2a^2-x^2}+b\sqrt{2a-x}}{\sqrt{2a^2-x^2}-b\sqrt{2a-x}}=\frac{\sqrt{a}+b}{\sqrt{a}-b},$$

(iii) $$\begin{cases} 27x+2y+3z=40, \\ x-y+z=3, \\ 2x+4y-3z=12. \end{cases}$$

4. The sum of a G.P., whose first term is 1, common ratio 3, and number of terms 4, is equal to the sum of an A. P., whose first term is 4, and common difference 4; how many terms are there in the A. P.?

5. If $a \propto b$, shew that

$$(a^2-b^2)(a^3+b^3) \propto (a-b)(a^4+b^4).$$

6. If m denote the number of combinations of n things taken two together, prove that the number of combinations of m things taken two together is equal to three times the number of combinations of $n+1$ things taken four together.

7. Transform 243 and 75 into the scale whose radix is 8; multiply them together in that scale and extract the square root of the result.

8. At a boat-race from Battersea to Putney it was observed that during the first mile and a quarter the crews of the first two boats pulled uniformly; during the remainder of the course, the winning boat gained one foot in forty-four; given that the number of miles from Battersea to Putney is three-fourths of the number of boats'-lengths by which the race was won, and that a boat is 40 feet long, find the distance from Battersea to Putney.

EXERCISE LXXX.

1. Divide

$$\frac{1}{10}a^{\frac{7}{10}} - \frac{1}{15}a^{\frac{8}{15}} + \frac{1}{20}a^{\frac{9}{20}} - \frac{1}{12}a^{\frac{2}{3}} + \frac{1}{18}a^{\frac{1}{2}} - \frac{1}{24}a^{\frac{5}{12}}$$

by

$$\frac{1}{5}\sqrt[5]{a} - \frac{1}{6}\sqrt[6]{a}.$$

2. Simplify

(i) $$\frac{6x}{3x-2} - \frac{30x^2+4x}{9x^2+4} + \frac{4x}{3x+2},$$

(ii) $$\frac{2x^3-7x^2y+2xy^2+3y^3}{6x^3-x^2y-4xy^2-y^3},$$

(iii) $$\frac{a^{\frac{5}{6}}+a^{\frac{1}{3}}b^{\frac{1}{3}}-a^{\frac{1}{2}}b^{\frac{1}{2}}-b^{\frac{5}{6}}}{a^{\frac{2}{3}}-b}.$$

3. Solve the equations

(i) $\dfrac{(1+x)^4}{1+x^4}=1,$

(ii) $\begin{cases} 4(x^2+y^2)=17(x+y), \\ \qquad xy=15. \end{cases}$

4. Simplify

$$\frac{2\sqrt{3}\,(1+\sqrt{3}+\sqrt{6})}{(\sqrt{2}+\sqrt{3})(\sqrt{3}+\sqrt{6})(\sqrt{6}+\sqrt{2})},$$

and extract the square root of $7\sqrt{5}\pm 2\sqrt{30}$.

5. Sum the series

(i) $\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{6}+\ldots$to 7 terms,

(ii) $\dfrac{3}{2}+1+\dfrac{2}{3}+\ldots$to 10 terms and to infinity,

(iii) $\dfrac{1}{2}+\dfrac{2}{2^2}+\dfrac{3}{2^3}+\dfrac{4}{2^4}+\ldots$to infinity.

6. If $a:b::c:d$, prove that

$$a^2+c^2 : b^2+d^2 :: \sqrt{a^4+c^4} : \sqrt{b^4+d^4}.$$

7. Write down the first five terms of the expansion of $(1-7x)^{\frac{1}{7}}$, and give in their simplest forms the middle terms of $(a^2-x^2)^{2n+1}$.

8. A farmer bought 100 animals, consisting of oxen, sheep, and ducks, for £100. If the oxen cost £5, the sheep £1, and the ducks 1*s*. each, how many of each had he?

Of how many solutions does this problem admit?

EXERCISE LXXXI.

1. If $x+y+z=2a$, and $x^2+xy+y^2+a^2=2a(x+y)$, shew that $(x-a)^2+(y-a)^2+(z-a)^2=a^2$.

2. Solve the equations

$$\text{(i)}\quad \frac{\dfrac{x+a}{x-a}+\dfrac{x-a}{x+a}}{\dfrac{x+b}{x-b}+\dfrac{x-b}{x+b}}=\frac{\dfrac{x+a}{x-a}+\dfrac{x+b}{x-b}}{\dfrac{x-a}{x+a}+\dfrac{x-b}{x+b}},$$

$$\text{(ii)}\quad \begin{cases} \dfrac{3y-7}{4}-\dfrac{2x+6}{5}+1\frac{1}{2}=0, \\[2ex] \dfrac{15y+1}{7}-\dfrac{5x+7}{2}+22\frac{2}{7}=0. \end{cases}$$

3. Resolve $1-a^{\frac{1}{2}}-a+a^{\frac{3}{2}}$ into factors, and prove that it is always positive when a is so.

Find the value of x which will give $2-4x-4x^2$ its greatest possible value.

4. The squares of the times of revolution of different planets vary as the cubes of their mean distances from the sun; compare the mean distances of two planets, whose times of revolution are respectively P and p.

5. Out of a dozen friends, how many different parties can be made consisting of *not more* than eight?

6. Of the series of weights $1, 2, 2^2, 2^3, \ldots$ lbs., find which must be selected to weigh 1719 lbs. Transform 40234 from the quinary to the duodenary scale.

7. Employ the Binomial Theorem to find the compound interest on £100, for 5 years, at 10 per cent.

8. A gentleman sends a lad into the market to purchase a shilling's worth of oranges. The lad having eaten

a couple, the gentleman pays at the rate of a penny for fifteen more than the market price. How many did the gentleman get for his shilling?

EXERCISE LXXXII.

1. Find the G.C.M. of

$$np^3q + 3np^2q^2 - 2npq^3 - 2nq^4$$

and $$2mp^2q^2 - 4mp^4 - mp^3q + 3mpq^3.$$

2. Simplify

$$4\sqrt{48} - 2\sqrt{98} + 11\sqrt{243} - 10\sqrt{300} + 7\sqrt{8},$$

and rationalize the denominators of

(i) $$\frac{43}{3\sqrt{5} - \sqrt{2}},$$

(ii) $$\frac{2x(\sqrt{a+x} + \sqrt{a-x})}{\sqrt{a+x} - \sqrt{a-x}}.$$

3. Solve the equations

(i) $$\frac{2}{x + \sqrt{2-x^2}} + \frac{2}{x - \sqrt{2-x^2}} = x,$$

(ii) $$\begin{cases} x + y = a, \\ x^4 + y^4 = 14x^2y^2. \end{cases}$$

4. A owes B 4*s.* 10*d.*; if A has only sixpences in his pocket, and B only fourpenny-pieces, how can they most simply settle the matter?

5. If $$\frac{2x-y}{2a+b} = \frac{2y-z}{2b+c} = \frac{2z-x}{2c+a},$$

shew that

$$21(a+b+c)(x+2y+3z) = (41a + 38b + 47c)(x+y+z).$$

6. If P be the p^{th} term and Q the q^{th} term of a G. P., shew that the n^{th} term

$$=\left(\frac{P^{n-q}}{Q^{n-p}}\right)^{\frac{1}{p-q}}.$$

If the p^{th}, q^{th}, r^{th}, s^{th} terms of an arithmetic progression be in geometrical progression, prove that $p-q$, $q-r$, $r-s$, are in geometrical progression.

7. Shew that if n be any whole number $n(n^2-1)(n^2-4)$ is divisible by 120.

8. Expand to 4 terms

(i) $\left(1-\dfrac{x}{2}\right)^{-\frac{5}{2}}$,

(ii) $\dfrac{1}{\left(2-\dfrac{x}{2}\right)^4}$,

(iii) $\dfrac{1}{\sqrt[5]{1-5x}}$.

Find by the Binomial Theorem $\sqrt[3]{120}$ to five places of decimals.

Exercise LXXXIII.

1. Having given that

$$\frac{b-a}{b+a}=\frac{4a-b}{6a-b},$$

find the ratio $a:b$; and if

$$\frac{c+d}{3d}=\frac{6c-d}{8c},$$

find the ratio $c:d$.

2. Solve the equations

(i) $$\frac{mx-(a+b)}{nx-(c+d)}=\frac{mx-(a+c)}{nx-(b+d)},$$

(ii) $$\begin{cases}\left(3+\frac{x}{y}\right)\left(3-\frac{y}{x}\right)=\left(3+\frac{y}{x}\right)\left(3-\frac{x}{y}\right),\\ (x-3)(y+3)=(2x-y)^2.\end{cases}$$

3. Find x from the equation

$$a\,.\,\frac{x+a}{x^2+a^2}=n,$$

and shew that the greatest value which the expression

$$a\,.\,\frac{x+a}{x^2+a^2}$$

admits of, for any value of x, is

$$\frac{\sqrt{2}+1}{2}.$$

4. If $a \propto \frac{1}{b}$, shew that $a^3b+a^2+\frac{a}{b} \propto \frac{1}{b^2}$.

There are two globes whose radii are r, r'; they are melted and formed into a single globe: given that the volume of a sphere varies as the cube of its radius, find the radius of the single globe.

5. If a, b, c be in geometrical progression, shew that $a+b$, $2b$, $b+c$ will be in harmonical progression.

6. The number of permutations of n things taken r together is to the number taken $r-1$ together as 10 to 1; and the corresponding combinations are as 5 to 3. Find n and r.

7. Transform 8978 from the scale of 11, and 3256 from the scale of 7, each to the scale of 12; and multiply the numbers together in that scale.

8. If a men or b boys can dig m acres of ground in

n days, shew that the number of boys whose assistance will be required to enable $a-p$ men to dig $m+p$ acres in $n-p$ days is

$$\frac{pb}{a}\left\{1+\frac{a(m+n)}{m(n-p)}\right\}.$$

Exercise LXXXIV.

1. Simplify the fraction

$$\frac{1}{\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x-2}},$$

and find its value when $x=2$.

Reduce to its simplest form

$$\frac{x^{\frac{3}{2}}y^{\frac{1}{2}}+xy+x+x^{\frac{1}{2}}yz^{\frac{1}{2}}+y^{\frac{1}{2}}z^{\frac{1}{2}}-1}{x^{\frac{3}{2}}y^{\frac{1}{2}}+xy+x-x^{\frac{1}{2}}yz^{\frac{1}{2}}-y^{\frac{1}{2}}z^{\frac{1}{2}}-1}.$$

2. Prove that

$$\frac{x^2+y^2+z^2}{yz+zx+xy} \text{ is } >2\,.\,\frac{bc+ca-ab}{a^2+b^2+c^2},$$

the first fraction being positive.

3. If $\dfrac{x}{y+z}=a$, $\dfrac{y}{z+x}=b$, $\dfrac{z}{x+y}=c$, shew that

$$\frac{x^2}{a(1-bc)}=\frac{y^2}{b(1-ca)}=\frac{z^2}{c(1-ab)},$$

and express each fraction in terms of x, y, z.

4. If $a+b : a-b :: b+c : 2(b-c) :: c+a : 3(c-a)$,

shew that $8a+9b+5c=0$.

5. Five boys from one school and four from another agree to form the crew of an eight-oared boat, on the condition that no two boys from the same school be seated together: how many different arrangements can be made of the crew so composed?

6. Expand $\dfrac{1}{\left(\sqrt[3]{\dfrac{1}{a}}-\sqrt[3]{\dfrac{1}{x}}\right)^6}$.

If s_1 be the sum of the odd terms of the expansion of $(a+x)^m$, and s_2 the sum of the even terms, then

$$s_1^2-s_2^2=(a^2-x^2)^m.$$

7. Solve the equations

(i) $$\frac{2x-3}{5}+\frac{1}{7}\left(5x-\frac{6x+4}{5x+1}\right)$$
$$=x+\frac{5x+8}{3x-14}+\frac{1}{3}\left(\frac{x}{7}+\frac{x-9}{5}\right),$$

(ii) $x^2+2\sqrt{x^2+2x+3}=12-2x$,

(iii) $\begin{cases} 2y^2=2x^2+1, \\ 2(y^2-1)=xy. \end{cases}$

8. A man bought a house which cost him 4 per cent. upon the purchase money to put into repair. It then stood empty for a year, during which time he reckoned he was losing 5 per cent. upon his total outlay. He then sold it again for £1192, by which means he gained 10 per cent. upon the original purchase money. What did he give for the house?

EXERCISE LXXXV.

1. Divide $x+y+z-3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$, and extract the square root of

$$\frac{x^2}{y^2}+\frac{y^2}{x^2}-\frac{1}{\sqrt{-1}}\left(\frac{x}{y}-\frac{y}{x}\right)-\frac{9}{4}.$$

2. Prove that

(i) $$\frac{1}{x(x-a)(x-b)}=\frac{1}{abx}+\frac{1}{a(a-b)(x-a)}$$
$$+\frac{1}{b(b-a)(x-b)},$$

(ii) $\sqrt{7+4\sqrt{3}}+\sqrt{7-4\sqrt{3}}=4.$

3. Solve the equations

(i) $\dfrac{6x+1}{15}-\dfrac{2x-4}{7x-16}=\dfrac{2x-1}{5}$,

(ii) $4x^3+6x^2+x=1$,

(iii) $\sqrt{2x+8}-2\sqrt{x+5}+2=0$.

4. Eliminate y and z from the equations

$$x+y+z=p,$$
$$xy+yz+zx=q,$$
$$xyz=r.$$

5. Sum the series

(i) $80+74+68+\ldots$ to 14 terms,

(ii) $1-\dfrac{2}{3}+\dfrac{3}{3^2}-\dfrac{4}{3^3}+\ldots$ to n terms.

6. Five ladies and three gentlemen are going to play at croquet: in how many ways can they divide themselves into sides of 4 each so that the gentlemen may not be all on one side?

7. Find the coefficient of x^{-15} in the expansion of $(1-x^{-3})^{-7}$, and prove that $4n+1$ is the coefficient of x^{2n} in the expansion of $\dfrac{1-x}{(1+x)^2}$.

8. If a man has only weights of 1, 3, 9, 27, ... lbs. each, find the smallest number of them with which he can weigh 475 lbs., and specify the number of each kind taken.

Exercise LXXXVI.

1. Find the L.C.M. of

$$x^5+ax^4+a^2x^3+a^3x^2+a^4x+a^5$$

and $x^5-ax^4+a^2x^3-a^3x^2+a^4x-a^5$.

2. If $\sqrt[4]{x}+\sqrt[4]{y}+\sqrt[4]{z}=0$, shew that
$$\{x^2+y^2+z^2-2(yz+zx+xy)\}^2=128xyz(x+y+z).$$

3. Solve the equations

(i) $\dfrac{x}{x+1}-\dfrac{3x}{x+2}+2=0$,

(ii) $\begin{cases} y(x^2+y^2)=a(x+y)^2, \\ \dfrac{xy}{a}=x+y, \end{cases}$

(iii) $2x+3y+4z=17$, in positive integers.

4. Find a number of three digits such that the sum of the first and second equals 7, the second and third 13, and of the third and first equals 8.

5. Sum the series

(i) $1+\dfrac{1}{3}-\dfrac{1}{3}-\ldots$ to 9 terms,

(ii) $2+4+10+\ldots$ to 6 terms.

If $$S_n=1+a+a^2+\ldots \text{ to } n \text{ terms},$$

and $$S_{-n}=1+\frac{1}{a}+\frac{1}{a^2}+\ldots \text{ to } n \text{ terms},$$

shew that $$\frac{S_n}{S_{-n}}=a^{n-1}.$$

Insert three harmonic means between 1 and 6.

6. If x varies directly as y and inversely as z, and 5, 6, and 7 be simultaneous values of x, y, z respectively, find the relation between y and z, and the value of z when $y=3$.

7. How many words can be made of the letters in the word "concatenation"?

The number of combinations of n things 3 together are 24 times as many as the combinations of $\frac{n}{2}$ things 4 together; find n.

8. The slope of a hill is 1 in 8. A man whose usual pace on level ground is 4 miles an hour, ascends and descends in 5 hours. His pace uphill : pace on level :: 3 : 4, and pace downhill to pace on level :: 5 : 4. What is the height of the hill?

Exercise LXXXVII.

1. Reduce to their simplest forms

(i) $$\frac{x^2+\left(\frac{a}{b}+\frac{b}{a}\right)xy+y^2}{x^2+\left(\frac{a}{b}-\frac{b}{a}\right)xy-y^2},$$

(ii) $$\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2-ca}{(b+a)(b+c)}+\frac{c^2-ab}{(c+a)(c+b)}.$$

2. Prove that the difference of the fractions

$$\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}} \text{ and } \frac{2\sqrt{2}+\sqrt{3}-1}{\sqrt{3}+1}$$

exceeds their product by 1.

3. Solve the equations

(i) $$\frac{(x+\cdot 5)(x+\cdot 75)}{(x+\cdot\dot{3})(x+\cdot 1\dot{6})}=\frac{x+1\cdot 25}{x+\cdot 5},$$

(ii) $$\begin{cases}\dfrac{1}{\sqrt{x+1}-\sqrt{x}}-\dfrac{1}{\sqrt{x+1}+\sqrt{x}}=y,\\ \sqrt{x}+\sqrt{y}=4.\end{cases}$$

4. If $x^2-yz=a^2$, $y^2-zx=b^2$, $z^2-xy=c^2$, shew that

$$a^2x+b^2y+c^2z-(a^2+b^2+c^2)(x+y+z)=0.$$

5. Find the product of the first 9 terms of the series

$$\frac{a^4}{b^4},\ \frac{a^3}{b^3},\ \frac{a^2}{b^2},\ldots.$$

Determine the geometrical mean between $a+b$ and

$$a^3-a^2b-ab^2+b^3.$$

The first four terms of a geometrical series are together equal to 45, and the first six terms to 189; find the series.

6. Find the coefficient of x^{20} in the expansion of

$$(x+x^2)^{12},$$

and of x^r in that of

$$(1+x)^{-\frac{10}{3}}.$$

7. Express 23·125 and 957 in the septenary scale, and the latter also in the scale whose radix is -7.

8. Out of a sphere of clay whose diameter is 8 feet, two spheres are formed with diameters 5 and 6 feet respectively; having given that the volume of a sphere varies as the cube of its radius, find the diameter of the sphere which can be formed of the clay which is left.

Exercise LXXXVIII.

1. Extract the square root of

(i) $ax^{-1}-2a^{\frac{1}{2}}x^{-\frac{1}{2}}+3-2a^{-\frac{1}{2}}x^{\frac{1}{2}}+a^{-1}x$,

(ii) $11-4\sqrt{7}$.

2. Simplify

(i) $\dfrac{x^{-3}-5a^{-1}x^{-2}-16a^{-2}x^{-1}+14a^{-3}}{x^{-3}-7a^{-1}x^{-2}-\ 2a^{-2}x^{-1}+14a^{-3}}$,

(ii) $\dfrac{a^3b-ab^3}{a^2+2ab+b^2}\div\dfrac{\frac{1}{b}-\frac{1}{a}}{a+b}$,

(iii) $\dfrac{(a^{-1}b^{-1}+c^{-1}d^{-1})^2-(a^{-1}c^{-1}+b^{-1}d^{-1})^2}{(a^{-2}-d^{-2})(b^{-2}-c^{-2})}$.

3. Solve the equations

(i) $\sqrt{\dfrac{x}{2}}+\sqrt{3x+1}=7,$

(ii) $\begin{cases} \dfrac{x+y}{x-y}=3, \\ x^2-y^2=3, \end{cases}$

(iii) $\begin{cases} y^4-12xy^2=432, \\ y^2-2xy=12. \end{cases}$

4. There is a number composed of two figures, of which the figure in the unit's place is triple of that in the ten's; if 36 be added to the number the sum is expressed by the same digits reversed. What is the number?

5. In a Geometrical Progression, if S_1, S_2, S_3 are the sums to n terms, $2n$ terms, $3n$ terms respectively, find the relation between S_1, S_2 and S_3.

6. In what numbers should 20 men be combined, so as to form the greatest possible numbers of different companies? In how many of these will the same pair of men be found together?

7. Shew that the difference between any number in the denary scale and that formed by reversing the digits is divisible by 9. Extend this theorem to any scale.

8. Expand $\dfrac{1}{\sqrt[3]{1-x^2}}$ to five terms, and if $x=1+h$ (where h is very small) shew that

$$\sqrt{\frac{2x-x^2+a^2x^2}{a^2+1}}=1+\frac{a^2h}{a^2+1} \text{ very nearly.}$$

Exercise LXXXIX.

1. Reduce to its simplest form

$$\frac{\dfrac{x^2+\sqrt{x^4-a^4}}{x^2-\sqrt{x^4-a^4}}-\dfrac{x^2-\sqrt{x^4-a^4}}{x^2+\sqrt{x^4-a^4}}}{4\sqrt{\dfrac{x^2-a^2}{x^2+a^2}}}.$$

2. Solve the equations

(i) $3x^n\sqrt[3]{x^n}+2\dfrac{x^n}{\sqrt[3]{x^n}}=16,$

(ii) $\begin{cases}\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=13,\\ \dfrac{1}{y}-\dfrac{1}{x}=1,\\ \dfrac{1}{xy}-\dfrac{2}{z}=0.\end{cases}$

3. If x be real, shew that the expression $x^2-8x+20$ can never be less than 4; and that $\dfrac{x^2-x+1}{x^2+x+1}$ lies between 3 and $\dfrac{1}{3}$.

4. If $x+y:3a-b::y+z:3b-c::z+x:3c-a$, prove that

$$x+y+z:ax+by+cz::a+b+c:a^2+b^2+c^2.$$

5. There are two cricket-clubs of 14 and 16 members respectively; how many different matches can they make, and in how many will two particular members, one out of each club, both play?

6. Expand $\left(\dfrac{a+x}{a-x}\right)^{\frac{2}{3}}$ in ascending powers of x, and find the coefficient of x^r in the expansion of $\left(x^{\frac{1}{2}}-\dfrac{1}{x^{\frac{1}{2}}}\right)^{2n}$.

7. A man saves £p a year from his income, which he invests annually at r per cent. compound interest: at the end of how many years will he have accumulated a capital, the interest of which is equal to his yearly saving?

8. There are two towns A and B; a traveller sets out from A towards B, and another from B towards A on the same day, and the travellers meet half way between A and B; one travels 5 miles the first day, 10 the second, 15 the third, and so on; the other travels 20 miles a day for two days, and 35 miles a day afterwards until they meet: find the number of days they travel, and the distance between A and B.

Exercise XC.

1. Shew that 10404 is a perfect square in every system of notation; and if $x+y$ be a multiple of p, and also $x-y$, then x^2-y^2 is a multiple of p^2.

2. Find the cube root of

$$\frac{a^6}{b^2c^2}-6a^4+12a^2b^2c^2-8b^4c^4.$$

3. Solve the equations

(i) $$\begin{cases} \dfrac{x}{3}+\dfrac{y}{2}=2+\dfrac{x+y}{5}, \\ \dfrac{x}{2}-\dfrac{y}{4}=3-\dfrac{x-y}{2}, \end{cases}$$

(ii) $(4+5x-x^2)^{\frac{1}{2}}=2^{\frac{3}{2}}x^{\frac{1}{2}}+(x^2+3x-4)^{\frac{1}{2}}$,

(iii) $3x+4y+7z=25$ (in positive integers).

4. A and B run a race; B has 50 yards' start, but A runs 20 yards while B runs 19; what must be the length of the course that A may come in a yard a head of B?

5. If $\dfrac{a+c}{b}=\dfrac{c}{a}=\dfrac{a}{c-b}$, determine the ratios $a:b:c$.

Also if $\dfrac{b}{a+b}=\dfrac{a+c-b}{b+c-a}=\dfrac{a+b+c}{2a+b+2c}$, determine the ratios $a:b:c$.

6. Sum the series

(i) $3+\dfrac{1}{2}-2-\ldots$ to 12 terms,

(ii) $3\frac{3}{8}+2\frac{1}{4}+1\frac{1}{2}+\ldots$ to 6 terms and to infinity.

Insert 3 harmonic means between 12 and $2\frac{2}{5}$.

There are m arithmetic means between 1 and 31 and the 7th : $(m-1)$th :: 5 : 9; shew that the number of means is 14.

7. Transform 7304·513 from the scale 8 to the scale 4; and multiply $3t4e2$ by $3t$ in scale 12.

8. The sides of a triangle are 141, 100, and 53 feet, find the length of the perpendicular on the longest side from the opposite angle.

EXERCISE XCI.

1. Find the G.C.M. of $x^{\frac{3}{2}}-3x+9x^{\frac{1}{2}}-27$

and $2x^2+23x+45$;

also the L.C.M. of

$$x^4+1,\ x^3+(\sqrt{2}-1)(x^2-x)-1, \text{ and } x^3+x^2-x-1.$$

2. Rationalize the denominator of the fraction

$$\frac{1}{\sqrt{a}-\sqrt{b}+\sqrt{c}},$$

and simplify

$$\frac{(a+b\sqrt{-1})^3+(a-b\sqrt{-1})^3}{(a+b\sqrt{-1})^2+(a-b\sqrt{-1})^2}.$$

3. Solve the equations

(i) $4x^3+6x^2+x=1$,

(ii) $\sqrt{x+1}+\sqrt{x+2}+\sqrt{x+3}+\sqrt{x+5}=0$,

(iii) $$\begin{cases} \dfrac{(x+y)^2}{a^2}+\dfrac{(x-y)^2}{b^2}=8, \\ x^2+y^2=2(a^2+b^2). \end{cases}$$

4. A person walking along a road in a thick fog, meets one waggon and overtakes another which is travelling at the same rate as the former: if a be the greatest distance to which he can see, and b, b' the distances which he walks between the times of his first seeing and passing the waggons, shew that a is an harmonic mean between b and b'.

5. If $x : b+c-a=y : c+a-b=z : a+b-c$,

shew that
$$(b-c)x+(c-a)y+(a-b)z=0,$$
and that
$$(a+b+c)\{x(y+z)+y(z+x)+z(x+y)\} \\ =2(x+y+z)(ax+by+cz).$$

6. Six papers are to be set in an examination, two of them in mathematics: in how many different orders can the papers be given, provided only that the two mathematical papers are not successive?

7. Prove that the coefficient of x^m in the expansion of $\dfrac{1}{(1-x)^{n+1}}$ is equal to the coefficient of x^n in the expansion of $\dfrac{1}{(1-x)^{m+1}}$.

Find the coefficient of x^r in the expansion of
$$\frac{(a+x)^{\frac{1}{2}}}{(a^2+x^2)^{\frac{1}{2}}(a-x)^{\frac{1}{2}}}.$$

8. Transform 1·69 to the scale of 4, extract its square root in that scale, and reduce the root to the scale of 10.

EXERCISE XCII.

1. Find the condition that x^2+ax+b and $x^2+a'x+b'$ may have a common measure of the form $x+c$.

2. A walks at the rate of 3 miles an hour, B at the rate of 1 mile an hour in a circular ring 3 miles round. B always proceeds in the same direction, but A turns back whenever he is within a mile of B. Supposing they start in the same direction from extremities of a diameter, prove that A will be in his original position and at a turning point after $3m \pm \frac{1}{2}$ hours, where m is any positive integer.

3. Solve the equations

(i) $(a\sqrt{x}+b)(b\sqrt{x}+a)=ab\sqrt{x}\left(\sqrt[4]{x}+\dfrac{c}{\sqrt[4]{x}}\right)^2$,

(ii) $\begin{cases} x^2-2xy=24, \\ xy-2y^2=\ \ 4, \end{cases}$

(iii) $xy+yz+zx=a^2-x^2=b^2-y^2=c^2-z^2$.

4. I owe a person a shilling and have nothing about me but sovereigns, and he has nothing but dollars worth 4*s*. 3*d*. each; how can I most simply discharge my debt?

5. Distinguish between interest and discount, and shew that, if P, I, D be respectively the principal sum, and the interest and discount upon it for any given time,

$$\frac{1}{D}=\frac{1}{I}+\frac{1}{P}.$$

6. The sums of the first n_1, n_2, n_3 terms of the same arithmetic series are S_1, S_2, S_3 respectively, prove that

$$\frac{S_1}{n_1}(n_2-n_3)+\frac{S_2}{n_2}(n_3-n_1)+\frac{S_3}{n_3}(n_1-n_2)=0.$$

The Arithmetic mean between two numbers exceeds the Geometric by 2, and the Geometric mean exceeds the Harmonic by $1\frac{13}{17}$: find the numbers.

7. How many permutations can be made of the letters of the word "Oolitic"?

Out of a party of twelve, how many sets of not more than five can be made up? How many times will any one person be chosen?

8. Find the greatest coefficient in the expansion of

$$\left(1+\frac{5x}{6}\right)^{\frac{3}{2}};$$

and prove that

$$\left(\frac{1+2x}{1+x}\right)^n=1+n\frac{x}{1+2x}+\frac{n(n+1)}{1.2}\left(\frac{x}{1+2x}\right)^2+\ldots$$

Exercise XCIII.

1. Prove that

$$\frac{a(b+c)}{(a-b)(c-a)}+\frac{b(c+a)}{(a-b)(b-c)}+\frac{c(a+b)}{(c-a)(b-c)}=1.$$

2. Solve the equations

(i) $\dfrac{\sqrt{3x+1}+\sqrt{3x}}{\sqrt{3x+1}-\sqrt{3x}}=4,$

(ii) $\begin{cases} ay^2+bxy-b=0, \\ bx^2+axy-a=0, \end{cases}$

(iii) $2x+3y+5z=17$ in positive integers.

3. Find the limits of value between which x must lie in order that $4x^2+4x-1$ may be positive: and if x be

real, prove that $\dfrac{2x-7}{2x^2-2x-5}$ can have no value between $\dfrac{1}{11}$ and 1.

4. Two particles, A and B, start simultaneously from the same point, and move in the same direction along the same straight line: A moves with the uniform speed of 2 feet per second, and the speed of B is such that it moves over 1 foot the first second, and that the number of feet described by it in any time varies as the square of the time. Prove that, during the motion, the particles will be three times at any given distance (less than a foot) from each other; and interpret the fourth result obtained in the algebraical solution of the problem.

5. If a, b, c, d be in Geometrical Progression, shew that

$$(a-b+c-d)^2=(a-b)^2+2(b-c)^2+(c-d)^2.$$

6. If $\dfrac{1}{x^2+x-6}$ be expanded in ascending powers of x, find the coefficient of x^8.

7. Find a fraction in the ternary scale equivalent to ·120120120... which is in the same scale.

8. On a sum of money borrowed, interest is to be paid at the rate of 5 per cent. per annum; after a time £200 of the loan is paid off, and the interest on the remainder reduced to 4 per cent.: the yearly interest paid is now lessened by one-third. What was the sum borrowed?

Exercise XCIV.

1. Shew that $p_0+p_1x+p_2x^2+\ldots+p_nx^n$ may be divided by $x+a$ with a remainder $p_0-p_1a+p_2a^2-\ldots\pm p_na^n$.

Hence determine when x^n+1 is divisible by $x+1$.

2. Solve the equations

(i) $$\begin{cases} xy + \dfrac{x}{y} = \dfrac{5}{3}, \\ \dfrac{1}{xy} + \dfrac{y}{x} = \dfrac{20}{3}, \end{cases}$$

(ii) $\sqrt{4a+x} + \sqrt{a+x} = 2\sqrt{2a+x}$,

(iii) $$\begin{cases} x^2 + y^2 + z^2 = 110, \\ x + y + z = 18, \\ x(y+z) = 65. \end{cases}$$

3. Eliminate x, y and z from the equations

$$x^2(y+z) = a^3,$$
$$y^2(z+x) = b^3,\ z^2(x+y) = c^3,$$
$$\text{and } abc = (x+y)(y+z)(z+x).$$

4. If $\dfrac{a}{b} = \dfrac{c}{d}$, shew that

$$\frac{\text{G.C.M. of } a+b \text{ and } a-b}{\text{G.C.M. of } c+d \text{ and } c-d} = \frac{\text{L.C.M. of } a+b \text{ and } a-b}{\text{L.C.M. of } c+d \text{ and } c-d}$$
$$= \sqrt{\frac{ab}{cd}}.$$

5. If 32 boys are drawn up in line 4 deep, in how many ways can they be arranged so as to have a different set in the front rank each time? In how many ways if the first rank is always to include the 3 tallest?

6. Sum the series $\dfrac{27}{16} + \dfrac{18}{15} + \dfrac{57}{80} + \dots$ to 6 terms.

If S_p denote the sum of the series $a + ar^p + ar^{2p} + \dots$ to infinity, and S_{-p} the sum of $a - ar^p + ar^{2p} - \dots$ to infinity, shew that

(*i*) $$S_2 S_{-2} = aS_4,$$

(ii) $\dfrac{1}{S_1}+\dfrac{1}{S_2}+\dfrac{1}{S_3}+\ldots\ldots+\dfrac{1}{S_n}=\dfrac{n}{a}-\dfrac{r}{a}\cdot\dfrac{r^n-1}{r-1}$.

7. Shew that 1030301 is a perfect cube in any scale of notation whose radix is >3, and that the square of every odd number, diminished by 1, is divisible by 8.

8. A stands in a line with 12 equidistant balls, and B, leaving A, picks up the balls in succession, returning with each to A, and when he has given him the last finds that he has walked exactly a mile. Had A and the balls all been equidistant, and the last ball been just as far from A as before, B would have had to walk 242 yards more. Find the distance of the balls from one another.

Exercise XCV.

1. Resolve into their elementary factors

(i) $a^2-b^2+c^2-d^2-2(ac-bd)$,

(ii) $(a^2-b^2)c+(b^2-c^2)a+(c^2-a^2)b$.

2. Simplify the expressions

(i) $\sqrt[3]{81}-\sqrt[3]{-512}+\sqrt[3]{192}-7\sqrt[6]{9}$,

(ii) $\sqrt[4]{97-56\sqrt{3}}$,

and find a multiplier that will rationalize $\sqrt[5]{3}+\sqrt[3]{5}$.

3. Solve the equations

(i) $\dfrac{2x-3}{2x-1}-\dfrac{2x-5}{2x-7}=1-\dfrac{4x^2-2}{4x^2-16x+7}$,

(ii) $x+\sqrt{1+x^2}=e^a$,

(iii) $4x^3-28x^2+61x-42=0$,

having given that the sum of two of the roots of (iii) is equal to the third.

4. If $a : b :: c : d :: e : f$, shew that

$$(ac+ce+ea)^2(b^2+d^2+f^2)$$
$$=(bc+de+fa)^2(a^2+c^2+e^2).$$

5. If a, b, c be positive and unequal numbers, prove that

(i) $\frac{a}{b}+\frac{b}{a}>2$,

(ii) $(a+b+c)(bc+ca+ab)>9abc$.

6. Shew that the expressions

$$(m^2-n^2-2mn)^2,\ (m^2+n^2)^2,\ \text{and}\ (m^2-n^2+2mn)^2$$

are in Arithmetical Progression; and find the values of m and n which will make them equal to 1, 25, 49, respectively.

7. In the expansion of $(1+x)^{-n}$, where both n and x are greater than unity, prove that the terms continually increase in numerical magnitude.

8. A is walking along a road and passes B; when finding he has lost something, he turns back and meets B t hours afterwards: having found what he had lost, he overtakes B again t' hours after he met him, and is T hours too late at his destination. Compare their rates of walking.

Exercise XCVI.

1. If $s=a+b+c+\ldots$ to n terms, shew that

$$\frac{s-a}{s}+\frac{s-b}{s}+\frac{s-c}{s}+\ldots=n-1.$$

2. Find the sum of n such fractions as

$$\frac{1}{1+x},\ \frac{2x}{1+x^2},\ \frac{4x^3}{1+x^4},\ \frac{8x^7}{1+x^8},\ \text{\&c.}$$

3. Solve the equations

(i) $\sqrt{3x-3}+\sqrt{5x-19}=\sqrt{3x+4}$,

(ii) $\begin{cases} x^4+y^4+2xy(x^2+y^2)=6x^2y^2, \\ x+y-xy=0. \end{cases}$

4. If the roots of the equation

$$ax^2+2bx+c=0$$

be possible and different, those of the equation

$$(a+c)(ax^2+2bx+c)=2(ac-b^2)(x^2+1)$$

will be impossible, and *vice versâ*.

5. If S_1, $S_2 \ldots S_p$ be the sums of infinite geometric series, whose first terms are the first p numbers in succession, and common ratios $\frac{1}{2}$, $\frac{1}{3} \ldots \ldots \frac{1}{p+1}$, prove that

$$S_1+S_2+\ldots S_p=(p+3)\frac{p}{2}.$$

6. If ${}_nC_r$ denote the number of combinations of n things r together, and $\frac{{}_{n+1}C_4}{{}_nC_2}=9$, find n; and if

$${}_nC_{r+1} : {}_{n+1}C_r : {}_{n-1}C_r :: 55 : 264 : 40,$$

find n and r.

7. Find the sum of the coefficients of powers of x in the expansion of $(p+x)^6$.

Find the cube root of 999 to 4 places of decimals.

8. Two coins of the same bulk, whose values are as 25 : 4, and whose weights are as 9 : 8, are each composed of silver and copper. Bulk for bulk silver is $\frac{1}{6}$ as heavy again as copper: weight for weight silver is 42 times as valuable as copper. Find the proportion of silver to copper in each coin.

EXERCISE XCVII.

1. Reduce the following expressions to their simplest forms

(i) $$\frac{\dfrac{\frac{1}{x}}{1+\frac{1}{x}}+\dfrac{1-\frac{1}{x}}{\frac{1}{x}}}{\dfrac{\frac{1}{x}}{1+\frac{1}{x}}-\dfrac{1-\frac{1}{x}}{\frac{1}{x}}},$$

(ii) $$\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-c)(x-a)}{(b-c)(b-a)}+\frac{(x-a)(x-b)}{(c-a)(c-b)}.$$

2. Solve the equations

(i) $$\left(\frac{x+a}{x+b}\right)^2+\left(\frac{x-a}{x-b}\right)^2=\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{x^2-a^2}{x^2-b^2}\right),$$

(ii) $$\begin{cases} x+y=\sqrt{5}+2, \\ \dfrac{x}{y}+\dfrac{y}{x}=\sqrt{5}, \end{cases}$$

(iii) $$\begin{cases} 3yz+2zx-4xy=16, \\ 2yz-3zx+\ \ xy=\ \ 5, \\ 4yz-\ \ zx-3xy=15. \end{cases}$$

3. Coaches start from A to B at 2 P.M. and from B to A at 4 A.M.: at noon the coach from B meets one from A, and again at 90 miles from A meets another. Find the distance between A and B and the rate of travelling.

4. Prove that

(i) $abc > (a+b-c)(b+c-a)(c+a-b)$,

(ii) $1.3.5\ldots(2n-1) < n^n$.

5. Eliminate x, y, z from the following equations

$$\lambda(x-a)+\mu(y-\beta)+\nu(z-\gamma)=1,$$
$$a'x^{\frac{1}{2}}+\beta'y^{\frac{1}{2}}+\gamma'z^{\frac{1}{2}}=\delta,$$
$$\frac{x}{a}=\frac{y}{\beta}=\frac{z}{\gamma}.$$

6. Sum the series $\frac{1}{2}-\frac{2}{3}-\frac{11}{6}-$ &c. to 10 terms.

The sum of 6 terms of the series $1-x\sqrt{-1}-x^2+\ldots$ is equal to 65 times the sum to infinity. Find x.

7. In the ordinary system of notation, shew that 12345679 multiplied by any multiple of nine less than 82 gives a row of the same digits. Prove the same property for the systems of notation in which the radices are 3, 4...8.

8. Write down the 5th term in the expansion of

$$(2+x)^7,$$

and find the greatest term when $x=3$.

What term of $(1-x)^{\frac{1}{2}}$ is equal to $\frac{1}{13}$ of the same term of $(1+x)^{-\frac{1}{2}}$?

Exercise XCVIII.

1. If $x^5-5qx+4r$ be divisible by $(x-c)^2$, then will $q^5=r^4$; and if ax^3+bx^2+cx+d be divisible by x^2+h^2, then will $ad=bc$.

2. Reduce to their simplest forms the expressions

(i) $$\frac{2(a^6+2b)-7a^{\frac{3}{2}}b^{\frac{1}{4}}(a^3+2\sqrt{b})+15a^3\sqrt{b}}{2(a^6-b)+5a^{\frac{3}{2}}b^{\frac{1}{4}}(a^3+2\sqrt{b})-15a^3\sqrt{b}},$$

(ii) $$\frac{7-2\sqrt{10}}{\sqrt{7-2\sqrt{10}}+5-\sqrt{10}}.$$

3. Solve the equations

(i) $\sqrt{2}+1-\dfrac{1}{2^{\frac{1}{x}}-1}=0,$

(ii) $(4x+2)^4-(3x-1)^4=(2x+4)^4-(x-3)^4,$

(iii) $\begin{cases}(x+y)(x^3+y^3)=1216, \\ x^2+xy+y^2=\quad 49.\end{cases}$

4. Which is the greater of the two ratios,

$$a^2-ab+b^2 : a^2+ab+b^2, \text{ or } a^4-a^2b^2+b^4 : a^4+a^2b^2+b^4;$$

a and b being of like sign?

If $x+y+z \propto x+y-z$, and $x^2+y^2+z^2 \propto x^2+y^2-z^2$, shew that $x \propto z$ and $y \propto z$.

5. Sum the series $1.3.5+3.5.7+\ldots$ to n terms.

If m, n be any two numbers, g their geometric mean, a_1, h_1, a_2, h_2 the arithmetic and harmonic means between m, g and g, n respectively, prove that

$$a_1h_2=g^2=a_2h_1.$$

6. Determine the greatest possible number of combinations that can be formed of the letters of the alphabet. In how many of these will all the first ten letters appear?

7. A person having a life income of £A a year, sets apart such a portion of it for the purpose of assuring his life, as will secure to his family, at his death, a sum, which, when put out to interest at r per cent., will yield them an income equal to that which now remains to him: if the annual premium on a policy of £100 be £a, find his remaining available income.

If $A=1000$, $a=3$, and $r=5$, find the annual premium on the sum assured.

8. Two numbers end with the same digit, and when divided by 7, the quotient of each is the remainder of the *other;* the sum of these is also 7: find the numbers.

EXERCISE XCIX.

1. Reduce to its simplest form the expression

$$\frac{(ab)^{x+y}+a^y b^x-a^x b^y-1}{\left(\frac{b}{a}\right)^x (ab)^y+b^{x+y}\{(ab)^y-(ab)^{-x}\}-b^{2y}},$$

and shew that

$$\frac{(1+a)\sqrt{1+b^2}-(1+b)\sqrt{1+a^2}}{a-b}$$

$$=\frac{2(1-ab)}{(1+a)\sqrt{1+b^2}+(1+b)\sqrt{1+a^2}}.$$

2. Solve the equations

(i) $\begin{cases} x^{2n}-y^{2n}=a^2, \\ x^n-y^n \quad =b, \end{cases}$

(ii) $x^4-6x^3+23x^2-42x-72=0,$

(iii) $\begin{cases} x^2+4y+\sqrt{2x^2+6y+10}=y+19, \\ x^2+y^2=4y+1. \end{cases}$

3. If α and β are the roots of the equation

$$ax^2+bx+c=0,$$

shew that the equation whose roots are $\dfrac{\alpha^3}{\beta^2}$ and $\dfrac{\beta^3}{\alpha^2}$ is

$$a^3c^2x^2+(5a^2bc^2-5ab^3c+b^5)x+a^2c^3=0.$$

4. If a, b, c; b, c, a; or c, a, b be in arithmetic progression, then will

$$\frac{2}{9}(a+b+c)^3=a^2(b+c)+b^2(c+a)+c^2(a+b);$$

and if in geometric progression,

$$a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3.$$

8

5. Shew that the number of combinations of $2n$ things, n of which are alike, taken n together $=2^n$.

6. Shew that any recurring decimal can be expressed as a terminating decimal by altering the scale of notation. Ex. $\cdot\dot{3}\dot{6}$ and $\cdot6\dot{4}2857\dot{1}$.

7. Expand $\dfrac{1+x}{(1-x)^3}$ by the Binomial Theorem, finding the remainder after n terms.

Shew that

$$1+3\left(1+\frac{1}{n-\frac{1}{2}}\right)+5\left(1+\frac{1}{n-\frac{1}{2}}\right)^2+\ldots$$
$$+(2n-1)\left(1+\frac{1}{n-\frac{1}{2}}\right)^{n-1}=n.(2n-1).$$

8. The distance from London to Peterborough is 75 miles, and from Peterborough to Grantham 30 miles. An up ordinary train starts from Grantham at the same time that the down express starts from London. The up train is delayed 30 minutes at Peterborough, and then passes the express 10 minutes after leaving Peterborough. Again, a down ordinary train leaves London at the same time that the up express leaves Grantham; each train travelling at the same rate as the other ordinary and express respectively; but in this case, the express, delayed only 6 minutes at Peterborough, arrives at the point where the former trains met, when the ordinary train has only got two-fifths of the distance of that point from London. Find the speed of the trains.

Exercise C.

1. If $a_1+a_2+\ldots+a_n=\dfrac{ns}{2}$, prove that

$$(s-a_1)^2+(s-a_2)^2+\ldots+(s-a_n)^2=a_1^2+a_2^2+\ldots+a_n^2.$$

2. Required the value of y which will make

$$2(y^3+y^2)x^3+(11y^2-2y)x^2+(y^2+5y)x+5y-1,$$

and

$$2(y^2+y)x^2+(11y-2)x+4$$

have a common measure different from unity.

3. Solve the equations

(i) $$2\{(x-1)(x-2)\}^2+4x^2+12x+8 = (x-1)^4+(x-2)^4,$$

(ii) $$(x^2-4x+8)^{\frac{1}{2}}-(x^2-5x+4)^{\frac{1}{2}}=3x+12,$$

(iii) $$\begin{cases} x^2+y^2+z^2-12a^2=0, \\ yz+zx+xy+6a^2=0, \\ x+y+\dfrac{a^2}{2}-\dfrac{3a}{2}=0. \end{cases}$$

What is the least number which being divided by 28, 19 and 15, leaves remainders 13, 2 and 7 respectively?

4. If any number be divided into two parts, shew that the sum of the cubes of the parts is least when the parts are equal.

5. If $\dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}=\ldots$, shew that

$$\frac{x^2+a^2}{x+a}+\frac{y^2+b^2}{y+b}+\frac{z^2+c^2}{z+c}+\ldots = \frac{(x+y+z+\ldots)^2+(a+b+c+\ldots)^2}{(x+y+z+\ldots)+(a+b+c+\ldots)}.$$

6. If a_r denote the r^{th} term of an harmonic series, shew that

$$\frac{1}{a_1}=\frac{r-1}{a_{r-1}}-\frac{r-2}{a_r}.$$

Continue the series $\dfrac{10}{57}, \dfrac{5}{27}, \dfrac{10}{51}$ to two terms each way, and find the twentieth term.

8—2

7. If there be $n+1$ different sets of n things of which there are two kinds, and if the first set consist all of one kind, the second set consist of $n-1$ of one kind, and 1 of the other, the third of $n-2$ of one kind and 2 of the other, and so of the rest; then the sum of the numbers of permutations which can be formed by taking all in each set together will be 2^n.

8. A tradesman, whose shop is lighted with two sizes of gas-burners, finding the larger size the more economical, (for he reckons they give half as much light again as the smaller size, whilst they consume only twenty per cent. more gas), has all the smaller-sized burners removed, and finds that by adopting the other size he can have as much light as before, at a saving of three shillings in every guinea that his gas used to cost him. By his new arrangement he has even less outlay than he would have had had all his old burners been of the small size, whilst he gets, of course, more light than they would have given; and the difference of expense would have been a saving of as many shillings in the pound as the number of large burners which represent the additional light. Required the number of burners with which the shop was originally lighted.

ANSWERS.

I.

1. 10.
2. 0; $2x+2z$.
3. $3x^2-4xy+8xz+8yz-4y^2-3z^2$.
4. a^2+ax+x^2.

II.

1. $-6x^3+x^2y+2x^2z+xy^2-11xyz+12xz^2-7y^3+5y^2z+yz^2$;
$-6x^3-x^2y+2x^2z+xy^2-11xyz+8xz^2-7y^3+8y^2z+yz^2$.
2. (i) b, (ii) $2x^2-2xy+2y^2$.
3. $10x^4-9x^3y-13x^2y^2+6xy^3$; $2x^3-7x+3$.
4. $a^2+b^2+c^2+d^2$.

III.

1. $4x+3$; 0.
2. $2a^2+6b^2-5c^2-16ab+12bc+2ca$.
3. $x^4-p^2x^2+2pq^2x-q^4$; a^2+ab+b^2.
4. (i) $x=3$, (ii) $x=2$.

IV.

1. a; a^2+b^2; c^2.
2. $3a^4-4a^3b+6a^2b^2+4ab^3+3b^4$.
3. $x^2-2xy+y^2$.
4. (i) $x=21$, (ii) $x=13$.

V.

1. 89.
2. $11ax-3by+10cz$; $12ax-2by+11cz$.
3. $6x^5-17x^4+21x^3-17x^2+9x-2$; x^2+2x+3.
4. (i) $x=2\frac{1}{3}$, (ii) $x=1$.

VI.

1. $99x-561$.
2. $x^3+y^3+z^3-3xyz$.
3. a^2-ab+b^2; $x^4-x^3y+x^2y^2-xy^3+y^4$.
4. (i) $x=10$, (ii) $x=-7$.

VII.

1. $4x-3a$; 84.
2. $\frac{13}{12}a^2+\frac{5}{12}b^2+\frac{5}{12}c^2$; $\frac{25}{12}a^2-\frac{1}{12}b^2-\frac{7}{12}c^2$.
3. $x^4-2x^2y^2+y^4$; x^6-y^6.
4. $\frac{x^2}{25}-\frac{3x}{5}+9$; $1+\frac{4x}{a}+\frac{8x^2}{a^2}+\frac{16x^3}{a^3}+\frac{32x^4}{a^4}+\dots\dots$
5. (i) $x=11$, (ii) $x=5$.

VIII.

1. $a^2+b^2+c^2+d^2+2ac+2bd$; b.
2. (i) 0, (ii) $3a-6b+6c+6d$, 0.
3. $\frac{1}{4}x^2+\frac{25}{8}xy+\frac{3}{2}y^2-\frac{1}{2}x-6y$.
4. $a^2+b^2+c^2-ab-ac-bc$.
5. (i) $x=0$, (ii) $x=4$.

IX.

1. 6; 0.
2. a^2-b^2.

3. $a^4+a^3x-9a^2x^2-23ax^3-10x^4$; $a^{4r}-b^{4p}$.

4. $\frac{3}{5}ax-2x^2$.

5. (i) $x=2$, (ii) $x=\frac{ad}{bc}$.

X..

1. -3.
2. $2a-5b+3c$; $4a+13b$.
3. $x^{16}-y^{16}$.
4. $a^4-a^3b+a^2b^2-ab^3+b^4$; $x^3+x+\frac{1}{x}+\frac{1}{x^3}$.
5. (i) $x=8$, (ii) $x=5$.

XI.

1. (i) x^2-y^2, (ii) 0; $2\frac{1}{2}$.
2. $4a+\frac{3b}{2}+\frac{c}{3}$; $\frac{7a}{2}+2b+\frac{2c}{3}$.
3. $3x^4-2x^3y-9x^2y^2+6xy^3-8y^4$;
 $x^{4r}-\frac{1}{2}x^{2r}y^{2s}+\frac{y^{4s}}{16}$.
4. $a^3+3a^2+9a+27$; $1+\frac{x^2}{a^2}+\frac{x^4}{a^4}+\frac{x^6}{a^6}+\ldots\ldots$
5. (i) $x=8$, (ii) $x=1\frac{9}{10}$.

XII.

1. $24a$; $(a-b)x-(b-c)y$.
2. 21.
3. $2a^2-3ab+4b^2$.
5. (i) $x=17$, (ii) $x=7$.

XIII.

1. (i) 9, (ii) 14.
2. $4a^3+4b^3$; $a^3+a^2b^3+b^3$.
3. $\dfrac{x^4}{6}+\dfrac{5x^3}{36}-\dfrac{x^2}{3}+\dfrac{5x}{6}-1$;
   $x^4+4ax^3-2a^2x^2-12a^3x+9a^4$.
4. $ay^2-ab+by$; $\dfrac{3x^3}{2}-5x^2+\dfrac{1}{4}x+9$.
5. (i) $x=2$, (ii) $x=3\frac{4}{7}$.
6. 36, 12, 2.

XIV.

1. x^8+x^4+1; x^8-x^4+1.
2. $x^3-6x^2y+9x^2z+12xy^2-36xyz$
   $+27xz^2-8y^3+36y^2z-54yz^2+27z^3$;
   $16a^8-96a^7x+216a^6x^2-216a^5x^3+81a^4x^4$.
3. $\pm\dfrac{21a^{10}b^5}{10x^7y^3}$; $-\dfrac{4a^3b^2c}{5xy}$; $\pm\dfrac{5a^6b^3c}{3x^2y^4}$; $\pm\dfrac{2}{3}p^5q^4r^3s^2t$.
4. (i) $x=-\dfrac{11}{7}$, (ii) $x=17$, (iii) $x=\dfrac{a-b}{a+b}$.
5. 7 miles per hour.

XV.

1. $a+b+c$.
2. $9a^6-30a^5b+61a^4b^2-72a^3b^3+56a^2b^4-24ab^5+4b^6$;
   $\dfrac{x^3}{a^3}+\dfrac{3x}{a}+\dfrac{3a}{x}+\dfrac{a^3}{x^3}$.
3. x^3-2x^2-3x.
4. $4x^2y^2$.
5. (i) $x=7$, (ii) $x=4$.
6. 30, 5, 4, 1.

XVI.

1. (i) $2a-5b+3c$, (ii) 0.
2. (i) x^2+2x+3, (ii) $a+b+c+d$.
3. $3a-b$.
4. x^2+x+1.
5. (i) $x=\frac{3}{5}$, (ii) $x=12$.
6. 21 shillings.

XVII.

1. $9a^2x^6$, $-27a^3x^9$; $\pm 8a^9b^6c^{-3}$, $4a^6b^4c^{-2}$; 4.
2. For s read $2s$, and the value is

$$\frac{1}{16}(a+b+c)(b+c-a)(c+a-b)(a+b-c).$$

4. G.C.M. $x-2a$; L.C.M. $(x^3-4ax^2+5a^2x-2a^3)(x^2+2ax+2a^2)$.
5. (i) $x=4$, (ii) $x=5\frac{1}{2}$.
6. £300, £100.

XVIII.

1. 1; ac.
2. $24x^{-3}+26x^{-2}+9x^{-1}+1$.
3. $x-3+\frac{2a}{x}$.
4. $x-2$.
5. (i) $x=8$, (ii) $x=5$, (iii) $\begin{cases} x=5, \\ y=11. \end{cases}$
6. 400 inches.

XIX.

1. $a+2b+c$.
2. x^2-x-1.

4. $x = -\frac{11}{7}$,

5. 7 miles per hour.

1. $a+b$

2.

XXII.

1. $\ldots - 4x^2y^2 + 4xy^3 - y^4$;

2. $\ldots y - \frac{1}{2}a^2z + \frac{1}{2}axy - \frac{1}{2}axz - \frac{1}{4}ayz - \frac{1}{4}xyz$.

3. [illegible]

4. $\ldots 3x + 5$.

5. (i) $x = \frac{2}{\ldots + 1}$, (ii) $\frac{4a^2}{a^2 - b^2}$.

6. $\ldots x^2 + 3x + 4$.

7. (i) $x = 1$, (ii) $x = -3$ or 2,

(iii) $x = \frac{2ab}{a+b}$, $y = -\frac{2ab}{a-b}$.

8. A in 3 days, B in 6 days, C in 9 days.

9. $\frac{1}{\ldots}(a + \ldots$

XXIII.

1. $\ldots + ac - b^2 - bc$.

2. (i) $\frac{a+b-c-d}{a-b-c+d}$, (ii) 0.

3. (i) G.C.M. $x - 1$, L.C.M. $(x-1)(2x-1)(2x+1)(3x-2)$;
(ii) G.C.M. $x^2 + xy + y^2$, L.C.M. $(x^2 + xy + y^2)$
$(x^2 - xy + y^2)(x + y)$.

4. $x^2 - ax - b$; $2x - 3y$.

5. (i) $x = -\frac{3}{2}$, (ii) $x = 1$, (iii) $x = \frac{1}{3}$ or $-\frac{1}{2}$.

6. 240 sovereigns, 480 [illegible] 720 pence.

1. $(x+3)(x+4)$, [illegible] $(3x-2)(5x+1)$,
$(7x-1)(2x-5)$.

2. $x^2 - 2x + 1$ [illegible]

(i) $\frac{a-5}{a-\ldots}$ [illegible]

3. $2x^2+4x-3$.

4. $(3a-5b)(3a+5b)$, $(2a+3b)(4a^2-6ab+9b^2)$, $(2a-3b)(4a^2+6ab+9b^2)$, $(3a-4b)(3a+4b)(9a^2+16b^2)$.

5. (i) $x=-6$, (ii) $x=-5$ or -4,
(iii) $x=5$, $y=4$, $z=3$.

6. 400.

XX.

1. 9.

2. $(x+5)(x+4)$, $(x-5)(x-4)$, $(x+5)(x-4)$, $(x-5)(x+4)$; $16x^4-24x^3+36x^2-54x+81$.

3. G.C.M. $x+7y$; L.C.M. $(x^2+y^2)(x^6-y^6)$.

4. (i) $8a^3b^6c^9d^{\frac{1}{2}}$, (ii) $\dfrac{x-5}{x+1}$, (iii) $\dfrac{x^2-7x+10}{x-10}$.

5. (i) $x=\dfrac{2a^2+2b^2}{a+b}$, (ii) $x=11$, $y=7$,
(iii) $x=2$ or -3.

6. 11.

XXI.

1. 18; $\dfrac{x}{y}-\dfrac{2y}{x}$.

2. $x^2-x-\dfrac{1}{2}$.

3. G.C.M. $3x-1$; L.C.M. $(x^2-4)(4x^2-1)(x^2-2x+4)$.

4. (i) $\dfrac{b}{a^2-b^2}$, (ii) $\dfrac{b}{a}$, (iii) $\dfrac{x^3+x^2-2}{2x^2+2x+1}$.

5. (i) $x=-2$, (ii) $x=6$ or $2\frac{5}{7}$,
(iii) $x=3$, $y=4$, $z=-3$.

6. $\dfrac{mn(a-b)}{mn-m-n}$.

XXII.

1. $2x^4 - x^3y - 4x^2y^2 + 4xy^3 - y^4$;

$a^3 + a^2x + \frac{1}{2}a^2y - \frac{1}{2}a^2z + \frac{1}{2}axy - \frac{1}{2}axz - \frac{1}{4}ayz - \frac{1}{4}xyz.$

2. $2x^2 + 3x + 5$.

3. (i) $\frac{2}{x+1}$, (ii) $\frac{4a^2}{a^2-b^2}$.

4. $x^3 + 2x^2 + 3x + 4$.

5. (i) $x = 1$, (ii) $x = -3$ or 2,
(iii) $x = \frac{2ab}{a+b}$, $y = -\frac{2ab}{a-b}$.

6. A in 3 days, B in 6 days, C in 9 days.

XXIII.

1. $a^2 + ac - b^2 - bc$.

2. (i) $\frac{a+b-c-d}{a-b-c+d}$, (ii) 0.

3. (i) G.C.M. $x-1$, L.C.M. $(x-1)(2x-1)(2x+1)(3x-2)$;
(ii) G.C.M. x^2+xy+y^2, L.C.M. (x^2+xy+y^2)
$(x^2-xy+y^2)(x+y)$.

4. $x^2 - ax - b$; $2x - 3y$.

5. (i) $x = -\frac{3}{2}$, (ii) $x = 1$, (iii) $x = \frac{1}{3}$ or $-\frac{1}{2}$.

6. 240 sovereigns, 480 shillings, 720 pence.

XXIV.

1. $(x+3)(x+4)$, $(x+3)(x-4)$, $(3x-2)(5x+1)$,
$(7x-1)(2x-5)$.

2. $x^2 - 2x + 1$.

3. (i) $\frac{a-5}{a-3}$, (ii) $\frac{2ap}{a^2+p^2}$.

4. $4x^4+3x^2+2$; $4a-3b$.

5. (i) $x=17$, (ii) $x=4$ or $-\frac{1}{6}$,
(iii) $x=3$, $y=2$, $z=1$.

6. 4 bowled, 3 run out, 1 left in.

XXV.

1. $4a$; $2m+2n+2p$, 1.

2. $(7a+4b)(7a-4b)$, $(2x+3y)(4x^2-6xy+9y^2)$,
$(4a^2+9b^2)(2a+3b)(2a-3b)$, $8ax(a^2+x^2)$.

3. G.C.M. $3a-2b$;
L.C.M. $5(3a-2b)(4a+5b)(5a+4b)(a+b)$.

4. 0; $\frac{(x-2)(x-1)}{x^2}$.

5. (i) $x=2\frac{6}{7}$, (ii) $x=2$ or -7, (iii) $x=6$ or 0,
(iv) $x=5$, $y=2$.

6. 100.

XXVI.

1. $x^3+x^2y+xy^2+y^3$; $y-x$.

2. $z+1+\frac{1}{z}$.

3. (i) $\frac{a^2-4ab-b^2}{(a+b)^2(a-b)^2}$,
(ii) $\frac{3x^2+2ax-2bx+2cx-ab+ac-bc}{(x+a)(x-b)(x+c)}$, (iii) 1.

4. (i) $x=\frac{3}{2}$, (ii) $x=1\frac{1}{10}$ or -1,
(iii) $x=\frac{2ab}{a^2+b^2}\cdot c$, $y=\frac{a^2-b^2}{a^2+b^2}\cdot c$.

5. 23.

6. $(x+y+a-b)(x+y+b-a)$
$(x+a+b-y)(y+a+b-x)$.

XXVII.

1. $\frac{20}{3}x+\frac{7}{4}y+\frac{9}{5}z$; $\frac{17}{3}x+\frac{3}{4}y+\frac{32}{15}z$.

2. (i) G.C.M. $x+6$, L.C.M. $(x+6)(x+5)(x-5)(x-7)$,
(ii) G.C.M. x^2+xy+y^2, L.C.M. $(x+y)(x-y)^2$
(x^2+xy+y^2).

3. $\frac{x^2}{x^3+y^3}$; (i) 1, (ii) $\frac{x^2-y^2}{x^4-x^2y^2+y^4}$.

4. (i) $\frac{y}{2}-x+\frac{2x^2}{y}$, (ii) $\frac{2x^2-3xy+5y^2}{2x+y}$.

5. (i) $x=7$, (ii) $x=5$ or $\frac{7}{2}$, (iii) $x=4$, $y=6$.

6. 12.

XXVIII.

1. $a^2-3ab+b^2$.

2. G.C.M. $7x-2y$;
L.C.M. $(7x-2y)^2(7x+2y)(3x-2y)(3x+2y)$.

3. 4; $\frac{1}{(m-n)^2}$.

4. (i) $x=9$, (ii) $x=0$, -3 or 4,
(iii) $x=1\frac{23}{25}$, $y=8$.

5. 6 dozen of oranges, 21 of apples.

6. 450.

XXIX.

1. $x^{2p(q-1)}-y^{2q(p-1)}$; $\frac{a^2}{25}-\frac{ab}{15}+\frac{b^2}{9}$.

2. G.C.M. $3x-1$, L.C.M. $(2x-1)(3x-1)(4x+1)$;
(i) $(p-q)(p+q)(p^2+q^2)(p^2-pq+q^2)(p^2+pq+q^2)(p^4+q^4)$,
(ii) $(x+1)(x-1)(7x+2)(7x-2)$.

3. $x^2-3x+2a$; $5x-2$.

4. (i) 0, (ii) $\dfrac{a^2(a-2b)}{(a-b)(a^2-ab+b^2)}$.

5. (i) $x=\dfrac{3}{2}$, (ii) $x=4$ or 2,
(iii) $x=\dfrac{(b-d)(c-d)}{(a-b)(a-c)}$, $y=\dfrac{(a-d)(c-d)}{(b-a)(b-c)}$, $z=\dfrac{(a-d)(b-d)}{(c-a)(c-b)}$.

6. 30,000.

XXX.

1. 74.

2. 3, 2, ± 2.

3. G.C.M. $x^2+2xy-y^2$; L.C.M. $(x+y)^2(x-y)^3$.

4. (i) 2, (ii) $\dfrac{x+y}{x-y}$.

5. (i) $x=4$, (ii) $x=5$, $y=7$,
(iii) $x=3$ or $7\frac{3}{11}$.

6. 2*s*. 6*d*.

XXXI.

2. G.C.M. $3x-7$;
L.C.M. $(a^2+a+1)(a^2-a+1)(a^2+1)(a+1)(a-1)$,
$(x-1)(x-3)(x-5)(x-7)$.

3. $\dfrac{a-b}{a+b}$; $-\dfrac{1}{8}$.

4. $\frac{x^2}{3}+x+\frac{1}{2}$; $2a+\frac{b}{2}$.

5. (i) $x=11$ or -13, (ii) $x=4$, $y=3$, $z=2$,
(iii) $x=4$, $y=3$.

6. $5\frac{5}{11}$ minutes after 2.

XXXII.

2. $\frac{x^3}{a^3}-\frac{3x}{a}+\frac{3a}{x}-\frac{a^3}{x^3}$.

3. (i) $\frac{a^4+a^2b^2-ab^3}{b^4+b^2a^2-ba^3}$, (ii) $\frac{3x+2}{x(x+1)^3}$,
(iii) $\frac{ab-1}{a^2b-ab^2-a+2b}$.

4. $2x-3y+4z$.

5. (i) $x=0$, $\frac{b}{a}$, or $-\frac{d}{c}$, (ii) $x=\frac{ad-bc}{2d-3b}$, $y=\frac{ad-bc}{3a-2c}$,
(iii) $\begin{cases} x=3 \text{ or } 2, \\ y=2 \text{ or } 3. \end{cases}$

6. 45 and 30 miles per hour.

XXXIII.

1. $a'x+b'y+c'^2$.

2. G.C.M. $x-3$; L.C.M. $(3x-2)(2x+1)(7x-1)$.

3. (i) $x=0$, or $-\frac{1}{9}$, (ii) $x=3$,
(iii) $\begin{cases} x=7, \\ y=4, \end{cases}$ (iv) $\begin{cases} x=4, \\ y=3, \\ z=2. \end{cases}$

4. -7, 3, 10.

5. $\frac{y^2}{2}-xy+2x^2$, $\frac{a}{b}+\frac{b}{a}$.

6. One mile; half an hour.

XXXIV.

2. $2x^2+1.$

3. (i) 0, (ii) 1, (iii) $\dfrac{x^2+y^2}{x}.$

4. (i) $x=4\frac{11}{16}$, (ii) $x=5$ or $1\frac{1}{5}$, (iii) $\begin{cases} x=\pm 5, \\ y=\pm 2. \end{cases}$

5. (i) $x^3-4x^2-4x+16=0,$
   (ii) $x^4-(a+b+c)x^3+(ab+bc+ca)x^2-abcx=0.$

6. $5\frac{2}{3}$ days.

XXXV.

1. (i) 0. (ii) $\dfrac{b+3}{a(b^2-b+1)}.$

2. $ax+by+c^2,$

3. (i) $\dfrac{a-b}{a+4b}$, (ii) $\dfrac{x^2-2x+3}{2x^2+5x-3}.$

4. (i) $(a+b+c)(a+b-c)(a-b+c)(a-b-c).$
   (ii) $2m^2n(m+n).$

5. (i) $x=\pm 6.$ (ii) $\begin{cases} x=\pm 3 \text{ or } \pm\dfrac{5}{\sqrt{2}}, \\ y=\pm 2 \text{ or } \pm\dfrac{1}{\sqrt{2}}. \end{cases}$

(iii) $\begin{cases} x=\dfrac{a}{\sqrt[4]{abc}}, \\ y=\dfrac{b}{\sqrt[4]{abc}}, \\ z=\dfrac{c}{\sqrt[4]{abc}}. \end{cases}$

6. 56 acres.

XXXVI.

2. (i) $30(x-1)(x-2)(x-3)$, (ii) a^6-b^6.

3. (i) $\dfrac{a^2+b^2}{a}$, (ii) $2a\left(\dfrac{a^2+b^2}{a^2-b^2}\right)^2$,

(iii) $\dfrac{z^2-x^2-y^2}{x^2-(y-z)^2}$.

4. (i) $x=0$ or $-2\pm\sqrt{-1}$, (ii) $\begin{cases} x=8, \\ y=12\frac{1}{4}, \\ z=17, \\ t=19, \end{cases}$

(iii) $\begin{cases} x=\pm\dfrac{a}{\sqrt{3}}, \\ y=\pm\sqrt{3}\,.\,b. \end{cases}$

5. $135x^2-6x-1=0$.

6. $8d$.

XXXVII.

2. G.C.M. $a^2+3ab-5b^2$; L.C.M. $(x-1)^3(x+1)$.

3. $x-\dfrac{a}{2}-\dfrac{b}{2}$, $\dfrac{a}{b}-\dfrac{b}{a}$.

5. (i) $x=\pm a$, $\dfrac{a}{2}$ or $-2a$,

(ii) $x=0$ or $-6\pm\sqrt{-11}$,

(iii) $\begin{cases} x=1 \text{ or } \dfrac{53}{19}, \\ y=2 \text{ or } -\dfrac{47}{19}. \end{cases}$

6. $\dfrac{2}{95}$th of a mile behind, (the second boat winning).

9

XXXVIII.

1. (i) G.C.M. $x-y$, L.C.M. $(x-y)(2x+3y)(3x^2+2xy+y^2)$.
(ii) G.C.M. $a+2b$, L.C.M. $(a^2-b^2)(a+2b)$.

2. (i) $4a^2$, (ii) $\dfrac{2a-b}{a+b}$.

3. $\dfrac{a}{2}-\dfrac{b}{3}+\dfrac{c}{4}$; $a^4-8a^2b+16b^2-64d=0$.

4. (i) $x=0$, 5 or 6, (ii) $x=3\frac{1}{2}$,
(iii) $\begin{cases} x=3, \\ y=5, \\ z=-7, \end{cases}$ (iv) $\begin{cases} x=2 \text{ or } 3, \\ y=3 \text{ or } 2. \end{cases}$

5. (i) $x^3-3x^2-10x+24=0$,
(ii) $x^4-13x^2+36=0$,
(iii) $x^2-2ax+a^2-b^2=0$.

6. 60.

XXXIX.

1. $\dfrac{1}{a^2}-\dfrac{1}{c^2}$, -3.

2. (i) $2(a-c)(1-ac)$, (ii) $(x-a)(x-b)(x^2-ab)$.

3. G.C.M. $x+y$, L.C.M. $(x+y)^3(x^2+y^2)(x-y)$;
G.C.M. x^2+x+2.

4. (i) $\dfrac{1}{x-a}$, (ii) $\dfrac{ax+by}{ax-by}$, (iii) $\dfrac{xy}{x^2+y^2}$.

5. (i) $\begin{cases} x=7, \\ y=9, \end{cases}$

(ii) $\begin{cases} x=\pm 2 \text{ or } \pm\dfrac{13}{2}\sqrt{-2}, \\ y=\pm 3 \text{ or } \mp 5\sqrt{-2}, \end{cases}$

(iii) $x=1$ or $-\dfrac{1}{2}(1\pm\sqrt{-3})$.

6. $\begin{cases} 845 \text{ and } 3380, \\ 1690 \text{ and } 2535. \end{cases}$

XL.

1. (i) $\dfrac{y+x}{y-x}$, (ii) $2+x$; $(1+x)^{2n}$.

3. $(x+y)(y+z)(z+x)$.

4. $\dfrac{3pq-p^3}{q}$; $x^2-(p^2-2q)\left(1+\dfrac{1}{q^2}\right)x+\dfrac{(p^2-2q)^2}{q^2}=0$.

5. (i) $x=-a$ or $-b$, (ii) $\begin{cases} x=\pm 7, \\ y=\pm 3, \end{cases}$

(iii) $\begin{cases} x=-a \text{ or } b, \\ y=-b \text{ or } a, \\ z=a+b \text{ or } 0. \end{cases}$

6. In 15′, when B has gone twice round exactly.

XLI.

1. (i) $x^{\frac{3}{p}}-(a+b+c)x^{\frac{2}{p}}+(ab+bc+ca)x^{\frac{1}{p}}-abc$,

(ii) $x^p+x^{\frac{p}{2}}y^{\frac{q}{2}}+y^q$.

2. $1+\dfrac{1}{2}x-\dfrac{1}{8}x^2+\dfrac{1}{16}x^3-\dfrac{5}{128}x^4+\&c.$; $x=10$.

3. (i) $\dfrac{bc+1}{abc+a+c}$, (ii) 1, (iii) $\dfrac{a}{b}$.

4. (i) $x=3$, (ii) $\begin{cases} x=1, \\ y=1, \end{cases}$

(iii) $\begin{cases} x=\dfrac{ab+bc+ca}{a} \text{ or } 0, \\ y=\dfrac{ab+bc+ca}{b} \text{ or } 0, \\ z=\dfrac{ab+bc+ca}{c} \text{ or } 0. \end{cases}$

5. 24.

6. The second, in the proportion of 80 : 79.

XLII.

1. $12\sqrt[12]{2000}$, $\frac{1}{8}$, $\sqrt{3}$.

2. $x^{\frac{4y}{5}} + x^{\frac{3y}{5}}y^{\frac{x}{5}} + x^{\frac{2y}{5}}y^{\frac{2x}{5}} + x^{\frac{y}{5}}y^{\frac{3x}{5}} + y^{\frac{4x}{5}}$.

3. (i) $\frac{2}{(x^2-1)^2}$, (ii) $\frac{1+x}{x}$, (iii) $\frac{(x+y)(x^2+y^2)}{x-y}$.

4. $x=4c$.

5. (i) $x=2\frac{4}{5}$, (ii) $x=\frac{25}{36}$, (iii) $\begin{cases} x=4 \text{ or } 3. \\ y=3 \text{ or } 4. \end{cases}$

6. 7 and 5, or $6\sqrt{2}$ and $\sqrt{2}$.

XLIII.

1. $a^{-\frac{8}{5}}x^{\frac{7}{5}} - x^{\frac{4}{5}} - a^{\frac{4}{5}}$.

2. $\frac{8x^3+4xy^2}{x^4+x^2y^2+y^4}$; x.

3. G.C.M. $p+2q$;
L.C.M. $(x-2)(x-1)(x+1)(x+3)(x+4)$.

4. (i) $\frac{3}{2}\sqrt{10}$, (ii) $\frac{2}{3}\sqrt[3]{18}$,

(iii) $3\sqrt{2}+\sqrt{7}$; $\frac{1}{2}(\sqrt{2}+\sqrt{26})$.

5. (i) $x=4$ or 1, (ii) $x=7$,
(iii) $\begin{cases} x=2,\ 3 \text{ or } -3\pm\sqrt{3}, \\ y=3,\ 2 \text{ or } -3\mp\sqrt{3}. \end{cases}$

6. Sherry 40*s*., Claret 50*s*. a dozen.

XLIV.

1. $x^2-2x-1.$

2. $(2y-x+a)(y-2x-a)$;
 L.C.M. $(3x+2)(x-1)(4x^2-1)$.

4. (i) $x=1\pm 2\sqrt{2}$,

 (ii) $\begin{cases} x=6, \\ y=12, \end{cases}$ (iii) $\begin{cases} x=8 \text{ or } 2, \\ y=4, \\ z=2 \text{ or } 8. \end{cases}$

5. 1296.

6. (i) $ax^{\frac{1}{2}}+b^{\frac{3}{4}}x^{\frac{1}{2}}+c^{\frac{2}{3}}x^{\frac{1}{2}}$, (ii) $a^{\frac{1}{2}}b^{\frac{1}{4}}c^{\frac{1}{6}}$,

 (iii) $a^{\frac{1}{2}}b^2+ab^2+\dfrac{b^2}{a^{\frac{2}{5}}}+\dfrac{b^2}{a^{\frac{4}{5}}}$.

XLV.

2. $x^{\frac{3}{2}}-xy^{-\frac{1}{2}}+x^{\frac{1}{2}}y^{-1}-y^{-\frac{3}{2}}$.

3. $\dfrac{3x^2-xy}{2x^3-y^2}$.

4. (i) $x=0, 49$ or 196, (ii) $x=1$,

 (iii) $\begin{cases} x=\pm 4, & \pm 3\sqrt{2}, & \pm 3, & \pm 2\sqrt{2}, \\ y=\pm 3, & \pm 2\sqrt{2}, & \pm 2, & \pm \frac{3}{2}\sqrt{2}. \end{cases}$

5. A wins; 16800 yards.

6. $2\sqrt{15}+4\sqrt{5}+3\sqrt{6}+6\sqrt{2}$; $2+\sqrt{-3}$.

XLVI.

2. $a+b+c-\sqrt{2ab}$; x^2-ax-a^2.

3. $m=n^{\frac{1}{n-1}}$; 4·4494.

4. (i) $\dfrac{x^2+2x+3}{2x^3+3x^2+x-1}$,

 (ii) $\dfrac{1}{(a-b)^2}$, (iii) $\dfrac{26}{9}$.

5. (i) $x = \pm 3$ or $\pm 3\sqrt{-1}$, (ii) $x = 4$,

(iii) $\begin{cases} x = \pm 5 \text{ or } \pm \dfrac{40}{3}\sqrt{-1}, \\ y = \pm 4 \text{ or } \mp \dfrac{41}{3}\sqrt{-1}. \end{cases}$

6. $1\frac{5}{6}$ hours.

XLVII.

1. $\dfrac{1}{2}$.

2. $3x^2(2x^2+3x+5)$.

3. (i) $\dfrac{2x^3+4}{x^4-1}$, (ii) $2\sqrt{a^2-x^2}$, (iii) $3+\sqrt{5}$.

4. (i) $x=3$,

(ii) $x = \dfrac{3}{2}(-1 \pm \sqrt{5})$,

(iii) $\begin{cases} x = 625 \text{ or } 1, \\ y = 1 \text{ or } 625. \end{cases}$

XLVIII.

1. 0; $\dfrac{4}{3}$, 1, $\dfrac{3}{4}$; a^{m-1}, $2a$.

2. $x^{-2} - 3x^{-1}y^{-1} + \dfrac{1}{3}y^{-2}$; $x^{\frac{1}{2}} - y^{\frac{1}{2}}z^{\frac{1}{2}}$.

3. (i) $\dfrac{2x+36x^{\frac{1}{3}}y^{\frac{2}{3}}}{x-27y}$,

(ii) $2\sqrt{7}+4\sqrt{14}+\sqrt{5}+2\sqrt{10}$, (iii) $\sqrt{1-x}$.

4. $\dfrac{4}{15}$.

5. $\dfrac{14}{3}\sqrt{5}$; $\dfrac{1}{2}(\sqrt{2}+\sqrt{6})$.

6. (i) $x=-\cdot 7$ or $-4\cdot 1$,

(ii) $\begin{cases} x=\pm 2 \text{ or } \pm\frac{3}{2}\sqrt{2}, \\ y=\pm 1 \text{ or } \pm\frac{1}{2}\sqrt{2}, \end{cases}$

(iii) $x=\frac{9}{16}a$, (iv) $x=a\mp\frac{1}{a}$ is one root.

7. $a^2+b^2=h^2+k^2$.

XLIX.

1. $x^{\frac{1}{2}}-\frac{2}{3}x^{\frac{1}{3}}+\frac{1}{2}x^{\frac{1}{4}}$.

2. (i) $-y^2$, (ii) $\frac{(a-b)^2}{a^2+b^2}$,

(iii) $\frac{(a+b)^4}{(a^2+b^2)^2}$.

3. G.C.M. $x+7$;

L.C.M. $(x+7)(x-2)(x-5)(x-10)(x^2-7x+1)$.

4. $\frac{1}{2}(2-\sqrt{3})$; $23+6\sqrt{15}$.

5. (i) $x=\frac{a^2b}{a^2+b^2}$,

(ii) $x=\frac{m^2-n^2}{ma-nb}$, $y=\frac{m^2-n^2}{mb-na}$, (iii) $x=4$,

(iv) $x=3$, -2 or $\frac{1\pm\sqrt{-23}}{2}$.

6. $-\frac{b}{a^4c}(b^2-4ac)(b^2-ac)$.

7. 756 miles; 36 and 27 miles per hour respectively.

L.

1. $n^2 = n + 1$.

2. (i) $60\,xy^2(x^6 - y^6)$,
   (ii) $(3x-2)(2x+1)(7x-1)$.

3. (i) $\dfrac{x-3}{x^2+2x+1}$,

   (ii) $\dfrac{ac-bd}{ac+bd}$, (iii) $\dfrac{4}{3(x+1)}$.

4. $(2+\sqrt{3})(2\sqrt{3}+\sqrt{5})$; $abc^{\frac{1}{3}}$.

5. (i) $\begin{cases} x = \dfrac{a(a^2+ab+b^2)}{b(a+b)}, \\ y = -\dfrac{a^2}{a+b}, \end{cases}$

   (ii) $x = 0$ or $-\dfrac{3}{2}$, (iii) $x = 8$ or $4\frac{4}{17}$.

6. $(p+1)n$, $(pq-1)n$, $(q+1)n$.

7. 40*s*. a dozen.

LI.

1. $a^{m+n} - b^2 a^{m+n-2} x^2 + 2bc\, a^{m+n-3} x^3 - c^2 a^{m+n-4} x^4$.

2. (i) $\dfrac{b+a}{a(b-a)}$, (ii) $\dfrac{15+2\sqrt{2}}{6}$.

3. $\dfrac{abb'}{b+b'}$ miles.

5. (i) $x = 7$,

   (ii) $x = \pm 11$ or $\pm\sqrt{-\dfrac{356}{3}}$,

(iii) $\begin{cases} x=\dfrac{2bc}{b+c}, \\ y=\dfrac{2ca}{c+a}, \\ z=\dfrac{2ab}{a+b}. \end{cases}$

6. 75.

7. $m(a^4+b^4)+n(a^4-b^4)=0.$

LII.

1. $x^{2^{n-1}}+y^{2^{n-1}}$; $b^2+c^2=ac.$

2. $x^{-2}-\frac{1}{2}x^{-1}y^{-1}+y^{-2}$; $\frac{1}{3}a^{-1}-\frac{1}{2}b^{-1}.$

3. $\dfrac{1}{a^8-x^8}.$

4. $7\sqrt{2}-3.$

5. (i) $x^4-14x^2+1=0$,
(ii) $144x^4-25x^2+1=0$,

6. (i) $x=5.$

(ii) $\begin{cases} x=5,\ 4 \text{ or } 8\pm\frac{1}{2}\sqrt{-174}, \\ y=4,\ 5 \text{ or } 8\mp\frac{1}{2}\sqrt{-174}, \end{cases}$

(iii) $x=\dfrac{p}{qr},\quad y=\dfrac{q}{rp},\quad z=\dfrac{r}{pq},$

where

$$p=\frac{1}{2}\left(\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a^2}\right),\ q=\frac{1}{2}\left(\frac{1}{c^2}+\frac{1}{a^2}-\frac{1}{b^2}\right),$$
$$r=\frac{1}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}\right).$$

7. Original capital £2,000,000.

LIII.

2. $x-3$.

3. (i) a^3b^2c, (iii) $(xyz)^{\frac{m}{p}}$.

4. 8, $2-\sqrt{3}$.

5. (i) $x=3$ or 1, (ii) $x=\frac{289}{864}$.

(iii) $\begin{cases} x=0 \text{ or } \pm 2, \\ y=\pm 2 \text{ or } 0. \end{cases}$

6. $\frac{3\pm\sqrt{5}}{2}$; $\frac{1\pm\sqrt{5}}{2}$.

7. $20\frac{20}{23}'$ after 8 o'clock.

LIV.

1. $a^4b\sqrt{b}+\frac{5}{6}a^3b^3\sqrt{a}-a^3b^3\sqrt{b}$; $\sqrt{ab}-\sqrt{bc}$.

2. $2x^2-x+\frac{1}{\sqrt{2}}$; $x-\sqrt{-1}$.

3. (i) 1, (ii) $\frac{b}{a+b}$.

4. $a+b$. The numerical G.C.M. is 36.

5. (i) $x=3$, (ii) $x=7\frac{1}{5}$,

(iii) $x=2$ or $-\frac{1}{2}$ or $\frac{3\pm\sqrt{505}}{4}$.

6. $1\frac{1}{2}$ ft. and 1 ft.

LV.

2. $\frac{16x^2-5x-13}{(2+3x)^3}$; $-\frac{2x(1+x)}{(1-x-x^2)(1+x+x^2)}$.

3. (i) $4x(x^2+y^2)^2(x^2-y^2)(x^4-x^2y^2+y^4)$,

(ii) $12x^2(x-a)(x-2a)(x+2a)$.

4. (i) $2\sqrt{5}-\sqrt{2}$.
   (ii) $9\sqrt{3}+3\sqrt{6}+6\sqrt{2}+4$, (iii) 1.

5. (i) $x=3^6$ or $\left(-\frac{1}{7}\right)^6$, (ii) $x=\pm 5$,
   (iii) $\begin{cases} x=\pm 7, \\ y=\pm 5. \end{cases}$

6. 8 or 9.

7. $15\frac{3}{4}$ and 16 minutes.

LVI.

1. $2(a+b+c)$; $1\frac{29}{30}$.

2. $\dfrac{a(2a+5x)(2a+7x)(a+6x)}{x(a-x)(a^2+2ax+2x^2)}$.

4. (i) $x=4\frac{1}{2}$, (ii) $\begin{cases} x=5 \text{ or } 3, \\ y=3 \text{ or } 5, \end{cases}$

   (iii) $\begin{cases} x=\dfrac{2}{b+c-a}, \\ y=\dfrac{2}{c+a-b}, \\ z=\dfrac{2}{a+b-c}. \end{cases}$

5. $\pm 3\sqrt{15}$; x^2+ax+a^2.

6. $y^2=\dfrac{b^2}{a^2}(a^2-x^2)$.

7. 30×25 yards.

LVII.

1. $x^{-2r}-\dfrac{2}{3}x^{-r}y^{-p}+\dfrac{y^{-2p}}{9}$.

2. (i) b^2, (ii) $3x+1$, (iii) $\sqrt{3}-\sqrt{2}$.

3. (i) $x=5$ or $-\frac{5}{4}$, (ii) $x=\pm\sqrt{\frac{b^2+ab+bc+2}{2}}$

(iii) $\begin{cases} x=5, \\ y=4, \\ z=3. \end{cases}$

4. 32 dozen and 80 dozen.

5. $\frac{bc-ad}{c-d}$.

6. (i) -35, (ii) 6, (iii) $3\frac{3}{4}$; $5\frac{7}{10}$, $5\frac{9}{10}$, $6\frac{1}{10}$, 6

7. *A* 7 miles an hour; *B* 6 miles an hour.

LVIII.

1. $a^3+2a^{\frac{3}{2}}b^{\frac{3}{5}}-2a^{\frac{1}{2}}b^{\frac{9}{5}}-b^{\frac{12}{5}}$;

$a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{4}}+a^{\frac{1}{4}}b^{\frac{1}{2}}+b^{\frac{3}{4}}$.

2. (i) $2\sqrt{3}$. (ii) $\frac{3+2x-x^2}{x(1+2x-x^2)}$.

3. (i) $x=13\frac{1}{3}$ or $15\frac{1}{3}$. (ii) $\begin{cases} x=\pm 3 \text{ or } \pm 1, \\ y=\pm 1 \text{ or } \pm 3. \end{cases}$

(iii) $\begin{cases} x=7,\ 2, \\ y=1,\ 4, \end{cases}$ (iv) $\begin{cases} x=\pm\frac{1}{3}\sqrt{6}, \\ y=\pm\sqrt{6}, \\ z=96. \end{cases}$

4. Half past 11.

5. $z=x+2x^2$.

6. 36, 30, 45.

7. $d=5$; $n=9$.

LIX.

2. G.C.M. $(2a^3+3a^2b-ab^2+b^3)ab$.
 L.C.M. $(2a^5+3a^4b+a^3b^2+4a^2b^3-ab^4+b^5)(a^2-b^2)a^2b^2$.

3. (i) $x=19$, (ii) $x=\frac{9}{5}$,

 (iii) $\begin{cases} x=4,\ 3,\ -6\pm 2\sqrt{6}, \\ y=3,\ 4,\ -6\mp 2\sqrt{6}, \end{cases}$ (iv) $\begin{cases} x=3, \\ y=4. \end{cases}$

4. 19.

6. (i) -120, (ii) $2\frac{3}{4}$, $2\frac{2}{3}$.
 $\frac{3}{2}$, $\pm\sqrt{2}$, $\frac{4}{3}$, $\pm\frac{8\sqrt{2}}{9}$; $\frac{32}{27}$.

7. $89\frac{7}{8}$; a fall of $\frac{1}{8}$.

LX.

1. $3a-2b$; a^2-b^2; $(3a-2b)x^2+(a^2-b^2)x$.

2. $4x^3+3x^2y-2y^3$, $3+4\sqrt{-1}$.

3. (i) $\frac{2(x^3+1)}{x^4+x^2+1}$, (ii) $\frac{1}{a+b}$, (iii) $x-\sqrt{x^2-a^2}$.

4. (i) $x=-b$, (ii) $x=1$, or $\frac{1}{2}(-7\pm\sqrt{-23})$.

 (iii) $x=\frac{49\pm\sqrt{97}}{8}$.

6. 4 or 14. When the equation to find n has equal roots.

7. 2 hours.

LXI.

2. $\sqrt{\dfrac{a}{b}}$; $a+b$.

3. (i) $x=0$, $\dfrac{1}{4}$ or -2,

(ii) $\begin{cases} x=3 \text{ or } -\dfrac{1}{3}, \\ y=-6 \text{ or } \dfrac{2}{3}, \end{cases}$

(iii) $x=-1$, -1 or $-1\pm\sqrt{3}$,

(iv) $\begin{cases} x=1 \text{ or } 5, \\ y=11 \text{ or } 4. \end{cases}$

4. A goose 4*s*., a duck 3*s*., a chicken 2*s*.

6. (i) $58\frac{1}{2}$, (ii) $\dfrac{n-1}{2}$,

(iii) $\dfrac{481}{1536}$, $\dfrac{8}{21}$; $\dfrac{5}{2}-\dfrac{1}{2}+\dfrac{1}{10}-\ldots\ldots$

7. $2:1$.

LXII.

1. $x^2-xy+y^2+x+y+1$.

2. (i) $-27x^9y^3$, (ii) $\dfrac{x\sqrt{xy}}{4a}$,

(iii) $\dfrac{a^3+b^3}{b}$, (iv) $\sqrt{5}-\sqrt{3}$.

3. (i) $\begin{cases} x=\pm 5, \\ y=\pm 2, \end{cases}$

(ii) $x=7$, -1 or $3\pm 2\sqrt{2}$,

(iii) $\begin{cases} x=\dfrac{2abc}{ab+ac-bc}, \\ y=\dfrac{2abc}{ab+bc-ac}, \\ z=\dfrac{2abc}{ac+bc-ab}. \end{cases}$

4. $\dfrac{3pq-p^3}{q^3}$.

6. $1+4-3+6-\ldots\ldots$; 7; 4.

7. 64.

LXIII.

1. p^3-p^2-2p+1 is the G.C.M. of the two expressions, which must each $=0$.

2. $b(x+y)$.

3. (i) $\dfrac{2x}{y^2(x^2-y^2)}$, (ii) $\dfrac{1}{(x+1)(x^2+1)}$,

(iii) $x^{\frac{1}{3}}-x^{-\frac{1}{3}}$.

4. (i) $x=\dfrac{1}{3}$ or -2,

(ii) $\begin{cases} x=4,\ 3,\ \dfrac{-19\pm\sqrt{313}}{2}, \\ y=3,\ 4,\ \dfrac{-19\mp\sqrt{313}}{2}, \end{cases}$

(iii) $\begin{cases} x=9,\ 6,\ 3, \\ y=2,\ 4,\ 6. \end{cases}$

5. $x=\dfrac{2}{27}y^3$.

7. Area 60 sq. ft.
Diagonal 13 ft.

LXIV.

2. (i) 0, (ii) $5\sqrt{6}$, (iii) -230, $472\sqrt{-1}$.

3. (i) $x=0$ or $-\dfrac{5}{2}$, (ii) $x=9\sqrt{2}$,

(iii) $\begin{cases} x=1, \\ y=-2, \\ z=3. \end{cases}$

6. $3ab^2 - a^3 = 2c^3$.

7. 600.

LXV.

1. $x^2-(c+d)x+cd$; $x^{2^{n-1}}+1$.

2. (i) 1, (ii) $\dfrac{15x^2-52x+77}{(5-3x)^3}$.

3. (i) 1; or, if $a=b$, $2a$.

4. (i) $x=\dfrac{1}{9}(5\pm\sqrt{61})$, (ii) $x=13$.

(ii) $\begin{cases} x=6,\ 5,\ 4,\ 3, \\ y=2,\ 4,\ 6,\ 8, \\ z=4,\ 3,\ 2,\ 1. \end{cases}$

5. $\pm 3\frac{3}{4}$; 169; 1 : 11.

6. (i) $-1\frac{2}{21}$, (ii) $\dfrac{1}{10}\cdot\dfrac{2^{\frac{5}{2}}+5^{\frac{5}{2}}}{2^{\frac{1}{2}}+5^{\frac{1}{2}}}$; the series has finite limit.

(iii) $4(2^n-1)-3n$.

7. 450.

LXVI.

1. $\left(\dfrac{a-b}{a+b}\right)^2$.

2. (i) $\begin{cases} x=4, \\ y=3, \end{cases}$ (ii) $x=1,\ \dfrac{1}{2},\ -1\pm\dfrac{1}{2}\sqrt{2}$.

(iii) $\begin{cases} x=9 \text{ or } 3\sqrt[3]{3}, \\ y=1 \text{ or } \dfrac{3(\sqrt[3]{3}-1)^2}{4\sqrt[3]{3}}. \end{cases}$

3. $-\sqrt{10}(\sqrt{2}+\sqrt[3]{3})(4+2\sqrt[3]{9}+3\sqrt[3]{3})$; $\sqrt{10}-\dfrac{1}{2}\sqrt{2}$.

6. $a+\sqrt{a^2-b^2}$ and $a-\sqrt{a^2-b^2}$; $\frac{3}{11}$, $3\frac{2}{5}$.

7. *10 bought;* 5 burnt.

ANSWERS.

LXVII.

1. $(x+y-z)(x-y+z+1)$.

2. (i) $\dfrac{1}{a^2+b^2}$, (ii) $4a\sqrt{a^2-1}$, (iii) $\dfrac{a}{b}+1-\dfrac{b}{2a}$.

3. (i) $x=\dfrac{65}{66}$, (ii) $x=9$, (iii) $\begin{cases} x=10, \\ y=\ \ 7, \\ z=\ \ 3, \end{cases}$

(iv) $\begin{cases} x=11,\ 3, \\ y=\ \ 1,\ 6. \end{cases}$

4. $nx^2-2mx+m=0$.

5. 66 yds.

7. $\dfrac{1}{6}$, $\dfrac{1}{2}$, $\dfrac{3}{2}$.

LXVIII.

1. $\dfrac{a^{4n}}{16}-b^{-4m}$.

2. $\dfrac{x^3+x^2+x+2}{3x^3-2x^2+x-1}$; $\dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}$.

3. $5\cdot8284$; $3+\sqrt{6}$.

4. (i) $x=2\frac{1}{2}$, (ii) $x=4a$ or $\dfrac{a}{14}(-9\pm\sqrt{-3})$,

(iii) $\begin{cases} x=2a \text{ or } 0, \\ y=2a \text{ or } 0. \end{cases}$

5. $(a\pm\sqrt{a^2-1})(b\pm\sqrt{b^2-1})(c\pm\sqrt{c^2-1})=1$.

7. (i) -126, (ii) $\dfrac{2}{\sqrt{2}-1}$; 8 and 2.

LXIX.

1. $2(a^2+b^2+c^2-ab-bc-ca)$.

2. G.C.M. $3x^2-2$;

L.C.M. $(x^3-x^2-17x-15)(3x^5+3x^4-53x^3+43x^2+34x-$

3. (i) $\begin{cases} x=2, \\ y=4, \\ z=6, \\ u=10, \end{cases}$ (ii) $\begin{cases} x=0 \text{ or } a, \\ y=b \text{ or } 0, \end{cases}$

(iii) $x=a\pm\dfrac{2a\sqrt{b}}{b+1}$.

4. 24 shillings, 40 florins, 56 half-crowns.

6. $y=\dfrac{1}{a^2-b^2}\left\{(ap-bq)\,x+(aq-bp)\,\dfrac{ab}{x}\right\}$.

7. 2, 3, 4, 6, 12.

LXX.

1. $\dfrac{x}{2y^{\frac{1}{2}}}-x^{\frac{1}{4}}y^{\frac{1}{4}}-\dfrac{y}{x^{\frac{1}{2}}}$; $\pm x$.

2. (i) $\dfrac{2(1+x^4)}{x}$, (ii) $2xy+2yz+2zx-x^2-y^2$

3. (i) $x=\pm\dfrac{1}{2}(3\pm\sqrt{5})$, (ii) $\begin{cases} x=2\frac{2}{3}, \\ y=5\frac{1}{3}, \end{cases}$

(iii) $\begin{cases} x=y=z=-\dfrac{a}{2}\pm\dfrac{\sqrt{a^2-4c^2}}{2}, \text{ or} \\ x=a,\ a,\ \dfrac{c^2-a^2}{a}, \\ y=a,\ \dfrac{c^2-a^2}{a},\ a, \\ z=\dfrac{c^2-a^2}{a},\ a,\ a. \end{cases}$

6. 3 ways.

7. $8\frac{1}{4}$ miles, $9\frac{3}{4}$ miles, $71\frac{1}{2}$ miles.

LXXI.

1. $a(x^2-xy+y^2)-b(y^2+yz+z^2)-c(x^2+xz-z^2)$; -146.

2. $x^2+(2m-3)x-6m$.

3. $\frac{1}{2}\sqrt{2}+\frac{1}{3}\sqrt{3}$; $\quad \frac{1}{2}(\sqrt{2}+\sqrt{6})$.

4. (i) $x=8$, $\quad$ (ii) $\begin{cases} x=144,\ 4,\ (7\pm\sqrt{73})^2, \\ y=4,\ 144,\ (7\mp\sqrt{73})^2, \end{cases}$

(iii) $\begin{cases} x=\ 6, \\ y=14, \\ z=12. \end{cases}$

5. 45.

7. $5,\ \pm 4,\ 3\frac{1}{5},\ \pm 2\frac{14}{25},\ 2\frac{6}{125}$.

8. 6.

LXXII.

1. $x^{\frac{5}{2}}+x^{-\frac{5}{2}}+\frac{11}{6}\left(x^{\frac{3}{2}}+x^{-\frac{3}{2}}\right)+\frac{11}{6}\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)$;
$4y+4(y^2-1)x-4x^2y$.

2. G.C.M. $x-4$; L.C.M. $12x^2(x-a)(x-2a)(x+2a)$.

3. (i) $x=1\frac{2}{13}$, $\quad$ (ii) $\begin{cases} x=\pm 2, \\ y=\pm 3, \end{cases}$ $\quad$ (iii) $\begin{cases} x=4, \\ y=3. \end{cases}$

4. A in 462, B in 231, C in $115\frac{1}{2}$ days.

7. (i) 301, $\quad$ (ii) $-10\frac{1}{2}$, $\quad$ (iii) $14\frac{3}{8}$, $14\frac{1}{2}$; $\quad 2\frac{2}{5}$, 3, 4, 6.

8. $\dfrac{\lfloor\underline{48}}{\lfloor\underline{15}\,\lfloor\underline{33}}$; $\quad \dfrac{\lfloor\underline{47}}{\lfloor\underline{14}\,\lfloor\underline{33}}$.

LXXIII.

1. (i) x^3, (ii) x^2, (iii) 2.

2. (i) $x = \pm\sqrt{b^2 + \frac{b}{a}}$,

(ii) $\begin{cases} x = 4,\ 2,\ 3 \pm \sqrt{-13}, \\ y = 2,\ 4,\ 3 \mp \sqrt{-13}. \end{cases}$

3. 288 and 224.

4. (i) $(2 + \sqrt{3})(2\sqrt{3} + \sqrt{5})$,

(ii) $\dfrac{(\sqrt{2} + \sqrt{3})(\sqrt{3} + \sqrt{5})(\sqrt{5} + \sqrt{2})(\sqrt{2} + \sqrt{3} - \sqrt{5})^2}{24}$.

6. $(b + x)^2$, $(b + x)^2 - 2bx, \ldots\ldots (b + x)^2 - 2(r - 1)bx \ldots\ldots$

7. 3360; 75600.

8. $243a^5 - 810a^4b^2 + 1080a^3b - 720a^2b^3 + 240ab^4 - 32b^5$.

$1 + \frac{1}{3}x^2 + \frac{2}{9}x^4 + \frac{14}{81}x^6 + \frac{35}{243}x^8 + \ldots\ldots$

LXXIV.

1. $\dfrac{1}{(x + a)^m (x + b)^n}$.

2. $\dfrac{7x - 2y}{5x^2 - 3xy + 2y^2}$.

3. (i) $x = a$, or $-\dfrac{b^2 + c^2}{b + c}$, (ii) $x = \dfrac{1}{2}$,

(iii) $\begin{cases} x = 4,\ 3,\ \frac{1}{2}(17 \pm \sqrt{-239}), \\ y = 3,\ 4,\ \frac{1}{2}(17 \mp \sqrt{-239}), \\ y = 5,\ -5. \end{cases}$

4. *124.*

ANSWERS.

6. (i) 330, (ii) -26, (iii) $6\frac{31}{125}$, $6\frac{1}{4}$; 4, 3, $2\frac{2}{5}$, 2, $1\frac{5}{7}$.

7. $-\dfrac{6435}{128}$;

$$\left(\frac{1}{4}\right)^{\frac{1}{3}}\left\{1-x+\frac{5}{4}x^2-\frac{5}{3}x^3+\ldots\right\}.$$

8. 2 minutes.

LXXV.

1. $\dfrac{(1-x)(3-x^2+x^3+x^5)}{(1+x)(1+x^2)(1+x^3)}$.

2. (i) $x=0, -1, -1$, (ii) $\begin{cases} x=3, \\ y=5, \end{cases}$

(iii) $\begin{cases} x=7, \dfrac{5}{6}, \\ y=5, -\dfrac{7}{6}. \end{cases}$

3. (i) $\dfrac{5}{12}\sqrt{6}$, (ii) $\dfrac{x+\sqrt{x^2-a^2}}{a}$.

6. 495; 165.

7. $\dfrac{x^4}{16}-x^3y+6x^2y^2-16xy^3+16y^4$;

$-792.5^5.4^7.x^5y^7$; $\dfrac{1.3.5\ldots\ldots(2n-3)}{2^{n-1}\lfloor n-1}x^{n-1}$.

8. 40235; 7.

LXXVI.

1. $x^r+2x^{\frac{r}{2}}y^{2p}+9y^{4p}$.

2. $x^{\frac{1}{2}}y^{\frac{1}{4}}+2y^{\frac{1}{3}}+\dfrac{1}{x^{\frac{1}{2}}}$; $x^{\frac{1}{5}}-\dfrac{a^{\frac{1}{10}}}{3}$.

3. (i) $x=7$ or $3\frac{4}{15}$, (ii) $\begin{cases} x=0 \text{ or } a-b, \\ y=-a \text{ or } -b, \end{cases}$

(iii) $x=0$ or 2, (iv) $x=4$, $y=1$.

4. *A* sold 4 quarters at 42*s*., *B* sold 8 at 50*s*.

5. $x=4$.

6. $2\frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{10}$, ... ; (i) 0, (ii) $1\frac{307}{512}$, $1\frac{3}{5}$, (iii)

7. 665280 ; 55440.

8. 142 ; its square is 32314.

LXXVII.

1. $(x-2)(x-1)(x+1)(x+3)(x+4)$.

2. (i) $x=0$, a, or $\frac{1}{2}\left\{a \pm \sqrt{a^2-16a+16}\right\}$,

(ii) $x=a$ or ∞, (iii) $x=2$, $y=3$, $z=4$.

3. The ratio $1+x : 1-x$. In 240, 180, and 14 respectively.

4. $a^{q-r}.c^{p-q}=b^{p-r}$.

5. 56.

6. $x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6$

$(3x)^{-4}+8(3x)^{-5}y+40(3x)^{-6}y^2+160(3x)^{-7}y^3+560(3x)$

$2099520\,a^7b^3$; $\dfrac{x^3}{16}$.

7. 85 in the denary scale.

8. *A* wins; he gains 5 shillings, and runs 341 Greatest area 84^2 square yards.

LXXVIII.

2. (i) 0, (ii) $\frac{26}{9}$.

3. (i) $\begin{cases} x = \pm 5, \ \pm 4\sqrt{-1}, \ \pm\frac{7}{2}\sqrt{2}, \text{ or } \pm\frac{1}{2}\sqrt{-62}, \\ y = \pm 4, \ \pm 5\sqrt{-1}, \ \pm\frac{1}{2}\sqrt{62}, \text{ or } \pm\frac{7}{2}\sqrt{-2}, \end{cases}$

(ii) $x = \left(\frac{a}{2}\right)^{\frac{1}{2}}(36-a^2)^{\frac{1}{4}}$.

4. (i) $b=0$, (ii) a and c of opposite signs, (iii) a and c of same sign, b negative, (iv) a and c of same sign, b positive. In equation (i) both roots negative, in (ii) roots equal in magnitude, but of opposite sign.

5. 34; any two consecutive digits of which the second is the greater up to 89.

7. $n=10$.

8. $1-15x+90x^2-270x^3+405x^4-243x^5$;

$1+9x+54x^2+270x^3$;

$1+x+2x^2+\frac{14}{3}x^3$; 9·949.

LXXIX.

2. $\frac{1}{5}\frac{a^2}{bc}+\frac{3}{10}a+\frac{1}{10}bc$; $2a^2-3a+1$.

3. (i) $x=0$, (ii) $x=0$ or a, (iii) $x=74$, $y=-349$, $z=-420$.

4. 4.

7. 363; 113; 43461; 207.

8. $2\frac{1}{4}$ miles.

LXXX.

1. $\frac{1}{2}a^{\frac{1}{2}} - \frac{1}{3}a^{\frac{1}{3}} + \frac{1}{4}a^{\frac{1}{4}}$.

2. (i) $\frac{16x(15x+2)}{81x^4-16}$, (ii) $\frac{x-3y}{3x+y}$,

(iii) $\frac{a^{\frac{1}{2}}+b^{\frac{1}{3}}}{a^{\frac{1}{3}}+b^{\frac{1}{2}}}$.

3. (i) $x=0$ or $\frac{-3\pm\sqrt{-7}}{4}$,

(ii) $\begin{cases} x=5,\ 3 \text{ or } \frac{-15\pm7\sqrt{-15}}{8}, \\ y=3,\ 5 \text{ or } \frac{-15\mp7\sqrt{-15}}{8}, \end{cases}$

4. $\sqrt{6}+\sqrt{3}-2\sqrt{2}-1$; $\sqrt[4]{5}(1\pm\sqrt{6})$.

5. (i) 0, (ii) $4\frac{5537}{13122}$, $4\frac{1}{2}$, (iii) 2.

7. $1-x-3x^2-13x^3-65x^4$;

$(-1)^n \frac{\lfloor 2n+1}{\lfloor n \lfloor n+1} a^{2n+2}x^{2n}$, $(-1)^{n+1} \frac{\lfloor 2n+1}{\lfloor n \lfloor n+1} a^{2n}x^{2n+2}$.

8. 19 oxen, 1 sheep, 80 ducks; one solution.

LXXXI.

2. (i) $x=0$ or $\pm\sqrt{-ab}$, (ii) $x=17, y=11$.

3. $(1-a^{\frac{1}{2}})^2(1+a^{\frac{1}{2}})$; $x=-\frac{1}{2}$.

4. The mean distances are as $P^{\frac{2}{3}} : p^{\frac{2}{3}}$.

5. 3784.

6. $1, 2, 2^2, 2^4, 2^5, 2^7, 2^9, 2^{10}$; $15t1$.

7. £61. 1*s*. $0\frac{6}{25}$*d*.

8. 18.

LXXXII.

1. $p-q$.

2. $15\sqrt{3}$; (i) $3\sqrt{5}+\sqrt{2}$, (ii) $2(a+\sqrt{a^2-x^2})$.

3. (i) $x=0$ or $\pm\sqrt{3}$,

(ii) $$\begin{cases} x = \dfrac{a}{2}(1\pm\sqrt{3}) \text{ or } \dfrac{a}{2}\left(1\pm\dfrac{1}{\sqrt{3}}\right), \\ y = \dfrac{a}{2}(1\mp\sqrt{3}) \text{ or } \dfrac{a}{2}\left(1\mp\dfrac{1}{\sqrt{3}}\right). \end{cases}$$

4. *A* gives 11 sixpences, and receives 2 fourpenny pieces.

8. (i) $1+\dfrac{5}{4}x+\dfrac{35}{32}x^2+\dfrac{105}{128}x^3$,

(ii) $\dfrac{1}{16}+\dfrac{x}{16}+\dfrac{5x^2}{128}+\dfrac{5x^3}{256}$,

(iii) $1+x+3x^2+11x^3$; 4·94609.

LXXXIII.

1. $a : b=2 : 5$; $c : d=3 : 4$ or $1 : 2$.

2. (i) $x=\dfrac{a+b+c+d}{m+n}$,

(ii) $x=y=\infty$, $x=-y=\dfrac{3}{10}(1\pm3\sqrt{-1})$.

3. $x=\dfrac{a}{2n}\left(1\pm\sqrt{1+4n-4n^2}\right)$.

4. $(r^3+r'^3)^{\frac{1}{3}}$.

6. $n=15$, $r=6$.

7. $6t12$; 814; $475t968$.

LXXXIV.

1. $\dfrac{x(x-1)(x-2)}{3x^2-6x+2}$, 0; $\dfrac{x+x^{\frac{1}{2}}y^{\frac{1}{2}}+y^{\frac{1}{2}}z^{\frac{1}{2}}-1}{x+x^{\frac{1}{2}}y^{\frac{1}{2}}-y^{\frac{1}{2}}z^{\frac{1}{2}}-1}$.

3. $\dfrac{(x+y)(y+z)(z+x)}{x+y+z}$.

5. 2880.

6. $a^2+6a^{\frac{7}{3}}x^{-\frac{1}{3}}+21\,a^{\frac{8}{3}}x^{-\frac{2}{3}}+56\,a^3x^{-1}+\ldots$

7. (i) $x=0$ or $-\dfrac{243}{193}$,

   (ii) $x=-1\pm\sqrt{7}$ or $-1\pm\sqrt{23}$,

   (iii) $x=\pm\dfrac{1}{2}\sqrt{3\pm\sqrt{\dfrac{11}{3}}}$, $y=\pm\dfrac{1}{2}\sqrt{5\pm\sqrt{\dfrac{11}{3}}}$.

8. £1000.

LXXXV.

1. $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}-y^{\frac{1}{3}}z^{\frac{1}{3}}-z^{\frac{1}{3}}x^{\frac{1}{3}}$;

   $\dfrac{x}{y}-\dfrac{1}{2\sqrt{-1}}-\dfrac{y}{x}$.

3. (i) $x=-2$, (ii) $x=-1$ or $\dfrac{-1\pm\sqrt{5}}{4}$,

   (iii) $x=\pm 4$.

4. $x^3-px^2+qx-r=0$.

5. (i) 574, (ii) $\dfrac{9}{16}+(-1)^{n-1}\left\{\dfrac{1}{2^4\,.\,3^{n-2}}+\dfrac{n}{2^2\,.\,3^{n-1}}\right\}$.

6. 30.

7. 462.

8. If the weights are placed in the same scale, the number required will be 9, for

$$475 = 3^5 + 2 \cdot 3^4 + 2 \cdot 3^3 + 3^2 + 2 \cdot 3 + 1.$$

LXXXVI.

1. $x^6 - a^6$.

3. (i) $x = -\frac{4}{5}$, (ii) $x = y = 0$ or $2a$,

(iii) $x = 1$, $y = 1$, $z = 3$, and three other solutions.

4. 167.

5. (i) -15, (ii) 370; $\frac{24}{19}, \frac{12}{7}, \frac{24}{9}$.

6. $yz = 42$, $z = 14$.

7. $\frac{\lfloor 13}{3 \cdot 2^5}$; $n = 10$.

8. $2062\frac{1}{2}$ yards.

LXXXVII.

1. (i) $\frac{ax + by}{ax - by}$, (ii) 0.

3. (i) $x = -\frac{17}{46}$, (ii) $x = 4$, $y = 4$.

5. 1; $\pm(a^2 - b^2)$; 3, 6, 12,...... or -9, 18, -36,......

6. $\frac{\lfloor 12}{\lfloor 4 \lfloor 8}$, $\frac{10 \cdot 13 \cdot 16 \ldots (3r + 7)}{(-3)^r \lfloor r}$.

7. $32 \cdot \dot{0}\dot{6}$; 2535; 15645.

8. $\sqrt[3]{171}$ feet.

LXXXVIII.

1. (i) $a^{\frac{1}{2}} x^{-\frac{1}{2}} - 1 + a^{-\frac{1}{2}} x^{\frac{1}{2}}$, (ii) $\sqrt{7} - 2$.

2. (i) $\frac{a^2 + 2ax - x^2}{a^2 - 2x^2}$, (ii) $a^2 b^2$, (iii) 1.

3. (i) $x=8$, (ii) $\begin{cases} x=\pm 2, \\ y=\pm 1, \end{cases}$

(iii) $\begin{cases} x=2 \text{ or } 3^{\frac{1}{3}}-3^{\frac{2}{3}}, \\ y=6 \text{ or } -2\cdot 3^{\frac{2}{3}}. \end{cases}$

4. 26.

5. $S_1^2+S_2^2=S_1(S_2+S_3)$.

6. 10; $\dfrac{\lfloor\underline{18}}{\lfloor\underline{8}\,\lfloor\underline{10}}$.

7. $1+\frac{1}{3}x^2+\frac{2}{9}x^4+\frac{14}{81}x^6+\frac{35}{243}x^8+\ldots\ldots$

LXXXIX.

1. $\dfrac{x^2(x^2+a^2)}{a^4}$.

2. (i) $x=2^{\frac{3}{2n}}$ or $\left(-\frac{8}{3}\right)^{\frac{3}{2n}}$,

(ii) $\begin{cases} x=\frac{1}{3} \text{ or } -\frac{1}{8}, \\ y=\frac{1}{4} \text{ or } -\frac{1}{7}, \\ z=\frac{1}{6} \text{ or } \frac{1}{28}. \end{cases}$

5. $\dfrac{\lfloor\underline{14}}{\lfloor\underline{11}\,\lfloor\underline{3}}\times\dfrac{\lfloor\underline{16}}{\lfloor\underline{11}\,\lfloor\underline{5}}$; $\dfrac{\lfloor\underline{13}}{\lfloor\underline{10}\,\lfloor\underline{3}}\times\dfrac{\lfloor\underline{15}}{\lfloor\underline{10}\,\lfloor\underline{5}}$.

6. $1+\frac{4}{3}\frac{x}{a}+\frac{8}{9}\frac{x^2}{a^2}+\frac{68}{81}\frac{x^3}{a^3}+\frac{176}{243}\frac{x^4}{a^4}+\ldots\ldots$;

$(-1)^{n-r}\dfrac{\lfloor\underline{2n}}{\lfloor\underline{n-r}\,\lfloor\underline{n+r}}$.

7. The equation to determine n is $(1+r)^n = 101$.
8. 12 days; 780 miles.

XC.

2. $\dfrac{a^2}{b^{\frac{2}{3}}c^{\frac{2}{3}}} - 2b^{\frac{4}{3}}c^{\frac{4}{3}}$.

3. (i) $\begin{cases} x=6, \\ y=4, \end{cases}$ (ii) $x = 1,\ -4$ or $\dfrac{5 \pm \sqrt{41}}{2}$,

(iii) $\begin{cases} x = 1,\ 2, \\ y = 2,\ 3, \\ z = 2,\ 1. \end{cases}$

4. 1020 yards.
5. 2 : 3 : 4; 2 : 3 : 4.
6. (i) -129; (ii) $9\frac{17}{72}$, $10\frac{1}{8}$; 6, 4, 3.
7. 323010·22102; 129tt98.
8. 28 feet.

XCI.

1. G.C.M. $x+9$; L.C.M. $(x^4+1)(x-1)(x+1)^2$.

2. $\dfrac{(\sqrt{a}+\sqrt{b}+\sqrt{c})(\sqrt{a}+\sqrt{b}-\sqrt{c})(\sqrt{a}-\sqrt{b}-\sqrt{c})}{a^2+b^2+c^2-2ab-2bc-2ca}$;

$\dfrac{a^3-3ab^2}{a^2-b^2}$.

3. (i) -1 or $\dfrac{-1 \pm \sqrt{5}}{4}$; (ii) $x = -\dfrac{71}{120}$,

(iii) $\begin{cases} x = \pm a \pm b, \\ y = \pm a \mp b. \end{cases}$

6. 480.
7. If r is of the form $4s$ the coefficient is

$$\frac{1.3.5\ldots\ldots(2s-1)}{\lfloor s \,.\, 2^s} \frac{1}{a^{4s+1}},$$

if of the form $4s+1$,

$$\frac{1 \cdot 3 \cdot 5 \ldots\ldots (2s-1)}{\underline{|s} \cdot 2^s} \frac{1}{a^{4s+2}},$$

if of the form $4s-1$ or $4s+2$, 0.

8. In scale 4 the square root is $1 \cdot 1\dot{0}\dot{3}$.

XCII.

1. $(ab'-a'b)(a-a')+(b-b')^2=0$.

3. (i) $x=\left\{\dfrac{ab(c^2-1)}{a^2+b^2-2abc}\right\}^2$, (ii) $\begin{cases} x=\pm 6, \\ y=\pm 1. \end{cases}$

(iii) $$\begin{cases} x= \pm \dfrac{abc}{2}\left\{\dfrac{1}{b^2}+\dfrac{1}{c^2}-\dfrac{1}{a^2}\right\}, \\ y= \pm \dfrac{abc}{2}\left\{\dfrac{1}{c^2}+\dfrac{1}{a^2}-\dfrac{1}{b^2}\right\}, \\ z= \pm \dfrac{abc}{2}\left\{\dfrac{1}{a^2}+\dfrac{1}{b^2}-\dfrac{1}{c^2}\right\}. \end{cases}$$

4. Pay £6, and receive 28 dollars.

6. 9 and 25.

7. 1260; 1585; 562.

8. The second $\left(=\dfrac{5}{4}\right)$.

XCIII.

2. (i) $x=\dfrac{3}{16}$, (ii) $\begin{cases} x=\pm\sqrt{\dfrac{a}{2b}}, \\ y=\pm\sqrt{\dfrac{b}{2a}}. \end{cases}$

(iii) $\begin{cases} x=2, 3, \\ y=1, 2, \\ z=2, 1. \end{cases}$

3. x must not lie between $-\dfrac{\sqrt{2}+1}{2}$ and $\dfrac{\sqrt{2}-1}{2}$.

6. $$-\left\{\frac{1}{2}\cdot\frac{1}{3^9}-\frac{1}{2^2}\cdot\frac{1}{3^8}+\frac{1}{2^3}\cdot\frac{1}{3^7}-\frac{1}{2^4}\cdot\frac{1}{3^6}+\frac{1}{2^5}\cdot\frac{1}{3^5}\right.$$
$$\left.-\frac{1}{2^6}\cdot\frac{1}{3^4}+\frac{1}{2^7}\cdot\frac{1}{3^3}-\frac{1}{2^8}\cdot\frac{1}{3^2}+\frac{1}{2^9}\cdot\frac{1}{3}\right\}.$$

7. $\dfrac{120}{222}$.

8. £1200.

XCIV.

1. When n is odd.

2. (i) $\begin{cases} x=\pm\dfrac{1}{2}, \\ y=\pm 3 \text{ or } \pm\dfrac{1}{3}, \end{cases}$ (ii) $x=-\dfrac{7}{8}a$,

(iii) $\begin{cases} x=5, \quad 13, \\ y=6,\ 7,\ \dfrac{1}{2}(5\pm\sqrt{-143}), \\ z=7,\ 6,\ \dfrac{1}{2}(5\mp\sqrt{-143}). \end{cases}$

3. $a^3+b^3+c^3+abc=0$ or $a^3+b^3+c^3-3abc=0$.

5. $\dfrac{\lfloor 32}{\lfloor 8\ \lfloor 24}$, $\dfrac{\lfloor 29}{\lfloor 5\ \lfloor 24}$.

6. $2\frac{13}{16}$.

8. 44 feet.

XCV.

1. (i) $(a+b-c-d)(a-b-c+d)$,

(ii) $(a-b)(b-c)(c-a)$.

2. (i) 8, (ii) $2-\sqrt{3}$;

$3^{\frac{14}{5}}-3^{\frac{13}{5}}.5^{\frac{1}{3}}+3^{\frac{12}{5}}.5^{\frac{2}{3}}-\ldots\ldots-3^{\frac{1}{5}}.5^{\frac{13}{3}}+5^{\frac{14}{3}}$.

3. (i) $x=-\frac{7}{8}$, (ii) $x=\frac{e^a-e^{-a}}{2}$,

(iii) $x=2, \frac{3}{2}$ or $\frac{7}{2}$.

6. $m=\pm\frac{3}{\sqrt{2}}$, $n=\pm\frac{1}{\sqrt{2}}$.

8. A's rate : B's rate :: $t+t' : t+t'-T$.

XCVI.

2. $\frac{1}{1-x}-\frac{2^n . x^{2^n-1}}{1-x^{2^n}}$.

3. (i) $x=4$, (ii) $\begin{cases} x=0, 2 \text{ or } -1\pm\sqrt{3}, \\ y=0, 2 \text{ or } -1\mp\sqrt{3}. \end{cases}$

6. $n=11$; $n=11$, $r=7$.

7. $(p+1)^6-p^6$; 9·9966.

8. The proportions of silver to copper are 57 : 7 in the first coin, 1 : 7 in the second.

XCVII.

1. (i) $\frac{x^2}{2-x^2}$, (ii) 1.

2. (i) $x=\frac{1}{2}\left\{\pm(a+b)\pm\sqrt{a^2+6ab+b^2}\right\}$,

(ii) $\begin{cases} x=\frac{3+\sqrt{5}}{2} \text{ or } \frac{1+\sqrt{5}}{2}, \\ y=\frac{1+\sqrt{5}}{2} \text{ or } \frac{3+\sqrt{5}}{2}, \end{cases}$

(iii) $x=\pm 1$, $y=\pm 2$, $z=\pm 3$.

3. 270 miles; 9 miles per hour.

5. $(\alpha'\alpha^{\frac{1}{2}}+\beta'\beta^{\frac{1}{2}}+\gamma'\gamma^{\frac{1}{2}})^2(1+\lambda\alpha+\mu\beta+\nu\gamma)$
$$-\delta^2(\lambda\alpha+\mu\beta+\nu\gamma)=0.$$

6. $-\dfrac{95}{2}$; $x=2$.

8. $280x^4$, which is the greatest term when $x=3$. The eighth.

XCVIII.

2. (i) $\dfrac{a^3-2a^{\frac{3}{2}}b^{\frac{1}{4}}+4b^{\frac{1}{2}}}{a^3+4a^{\frac{3}{2}}b^{\frac{1}{4}}-2b^{\frac{1}{2}}}$, (ii) $\dfrac{1}{4}(\sqrt{5}-1)(\sqrt{5}-\sqrt{2})$.

3. (i) $x=2$, (ii) $x=\pm 1$ or $\dfrac{-3\pm\sqrt{5}}{2}$,

(iii) $\begin{cases} x=\pm 5,\ \pm 3 \text{ or } \dfrac{1}{4}(\pm\sqrt{38}\pm\sqrt{670}), \\ y=\pm 3,\ \pm 5 \text{ or } \dfrac{1}{4}(\pm\sqrt{38}\mp\sqrt{670}). \end{cases}$

4. The second is the greater.

5. $n(2n^3+8n^2+7n-2)$.

6. $\dfrac{\lfloor 26}{(\lfloor 13)^2}$; $\dfrac{\lfloor 16}{\lfloor 3 \lfloor 13}$.

7. £ $\dfrac{Ar}{a+r}$; £375.

8. 13 and 43.

XCIX.

1. $\dfrac{a^x}{b^y}$.

2. (i) $x=\left(\dfrac{a^2+b^2}{2b}\right)^{\frac{1}{n}}$, $y=\left(\dfrac{a^2-b^2}{2b}\right)^{\frac{1}{n}}$,

(ii) $x=4,\ -1$ or $\frac{3}{2}(1\pm\sqrt{-7})$,

(iii) $\begin{cases} x=\pm 1,\ \pm 2,\ \pm\sqrt{\frac{3}{2}}\,.\sqrt{11\pm\sqrt{-55}}\,, \\ y=4,\ 3,\ \frac{1}{2}(7\pm\sqrt{-55}). \end{cases}$

7. $1+3x+5x^2+\ldots+(2n-1)\,x^{n-1}$

$$+\frac{(2n+1)\,x^n-(2n-1)\,x^{n+1}}{(1-x)^2}.$$

8. 42 miles per hour, and 30 miles per hour.

C.

2. $y=5$.

3. (i) $x=\frac{1}{24}$,

(ii) $x=-4$ is one root,

(iii) $\begin{cases} x=\frac{1}{4}\left(3a-a^2\pm a\sqrt{3}\,\sqrt{23+6a-a^2}\right), \\ y=\frac{1}{4}\left(3a-a^2\mp a\sqrt{3}\,\sqrt{23+6a-a^2}\right), \\ z=\frac{1}{2}\left(a^2-3a\right); \end{cases}$

97.

6. $\frac{10}{63},\ \frac{1}{6},\ldots\ldots\frac{5}{24},\ \frac{2}{9};\ \infty$.

8. 1 large, 3 small burners.

CAMBRIDGE: PRINTED AT THE UNIVERSITY PRESS.

16, BEDFORD STREET, COVENT GARDEN, W.C.
London and Cambridge.

MACMILLAN AND CO.'S

DESCRIPTIVE CATALOGUE OF

CAMBRIDGE CLASS BOOKS.

The Works in this Series of CAMBRIDGE CLASS-BOOKS FOR THE USE OF SCHOOLS AND COLLEGES, *which have been issued at intervals during the last ten years, are intended to embrace all branches of Education, from the most Elementary to the most Advanced, and to keep pace with the latest discoveries in Science.*

Of those hitherto published the large and ever increasing sale is a sufficient indication of the manner in which they have been appreciated by the public.

A SERIES of a more Elementary character is in course of publication, a list of which will be found on page 2 of this Catalogue.

2000 :
2 : 27 : 65.

CAMBRIDGE CLASS BOOKS

FOR

SCHOOLS AND COLLEGES.

WORKS by the Rev. BARNARD SMITH, M.A.

Fellow of St. Peter's College, Cambridge.

1.

·ithmetic & Algebra

heir Principles and Applications.

h numerous Examples, systematically arranged.

th Edit. 696 pp. (1863). Cr. 8vo. ongly bound in cloth. 10*s.* 6*d.*

first edition of this work was published 1854. It was primarily intended for e use of students at the Universities, d for Schools which prepare for the ıiversities. It has however been found meet the requirements of a much rger class, and is now extensively used *Schools* and *Colleges* both *at home* and the *Colonies.* It has also been found great service for students preparing r the MIDDLE-CLASS and CIVIL AND ILITARY SERVICE EXAMINATIONS, from e care that has been taken to elucidate e *principles* of all the Rules. Testiony of its excellence has been borne by me of the highest practical and theotical authorities; of which the following from the late DEAN PEACOCK may taken as a specimen:

Mr. Smith's Work is a most useful lication. The Rules are stated with ıt clearness. The Examples are well cted and worked out with just suffit detail without being encumbered by minute explanations; and there pres throughout it that just proportion of ry and practice, which is the crownexcellence of an elementary work."

2. Arithmetic

For the Use of Schools.

New *Edition (1862)* 348 pp. *wn 8vo. strongly* bound in cloth, . *6d.* Answers to all the Quesns.

3. **Key** to the Arithmetic, containing Solutions to all the Questions in the latest Edition. Cr. 8vo. cloth. 392 pp. Third Edit. 8*s.* 6*d.*

To meet a widely expressed wish, the ARITHMETIC was published separately from the larger work in 1854, with so much alteration as was necessary to make it quite independent of the ALGEBRA. It has now a very large sale in all classes of Schools at home and in the Colonies. A copious collection of Examples, under each rule, has been embodied in the work in a systematic order, and a Collection of Miscellaneous Papers in all branches of Arithmetic is appended to the book.

4. **Exercises in Arithmetic.** 104 pp. Cr. 8vo. (1860) 2*s.* Or with **Answers,** 2*s.* 6*d.* Also sold separately in 2 Parts 1*s.* each. **Answers,** 6*d.*

These EXERCISES have been published in order to give the pupil examples in every rule of Arithmetic. The greater number have been carefully compiled from the latest University and School Examination Papers.

5. School Class Book of Arithmetic.

1864. 124 pp. PART I. to the end of Compound Division. 18mo. limp cloth. 10*d.*—PART II. 104 pp. containing Fractions, Decimals, Subtraction of Square and Cube Root. 18mo. limp cloth. 10*d.* —PART III. will contain Rule of Three, Interest. &c.

The School Class Book of Arithmetic, published at the request of many Schoolmasters, and chiefly intended for National and Elementary Schools, has been prepared on the same plan as that adopted in the Author's School Arithmetic, which in extensive circulation in England abroad.

WORKS by ISAAC TODHUNTER, M.A. F.R.S.

Fellow and Principal Mathematical Lecturer of St. John's College, Cambridge.

1. Algebra.

For the Use of Colleges and Schools.

Third Edition. 542 pp. (1862). Strongly bound in cloth. *7s. 6d.*

This work contains all the propositions which are usually included in elementary treatises on Algebra, and a large number of *Examples for Exercise.* The author has sought to render the work easily intelligible to students without impairing the accuracy of the demonstrations, or contracting the limits of the subject. The Examples have been selected with a view to illustrate every part of the subject, and as the number of them is about *Sixteen hundred and fifty*, it is hoped they will supply ample exercise for the student. Each set of Examples has been carefully arranged, commencing with very simple exercises, and proceeding gradually to those which are less obvious.

2. Plane Trigonometry

For Schools and Colleges.

2nd Edit. 279 pp. (1860). Crn. 8vo. Strongly bound in cloth. *5s.*

The design of this work has been to render the subject intelligible to beginners, and at the same time to afford the student the opportunity of obtaining all the information which he will require on this branch of Mathematics. Each chapter is followed by a set of Examples; those which are entitled *Miscellaneous Examples*, together with a few in some of the other sets, may be advantageously reserved by the student for exercise after he has made some progress in the subject. As the Text and Examples have been tested by considerable experience in teaching, the hope is entertained that they will be suitable for imparting a sound and comprehensive knowledge of Plane Trigonometry, together with readiness in the application of this knowledge to the solution of problems. In the Second Edition the hints for the solution of the Examples have been considerably ncreased.

3. Spherical Trigonometry.

For the Use of Colleges and Schools.

132 pp. Crown 8vo. Second Edit. enlarged. (1863). *4s. 6d.*

This work is constructed on the same plan as the *Treatise on Plane Trigonometry*, to which it is intended as a sequel. Considerable labour has been expended on the text in order to render it comprehensive and accurate, and the Examples, which have been chiefly selected from University and College Papers, have all been carefully verified.

The Elements of Euclid

For the Use of Schools and Colleges.

COMPRISING THE FIRST SIX BOOKS AND PORTIONS OF THE ELEVENTH AND TWELFTH BOOKS, WITH NOTES, APPENDIX, AND EXERCISES.

384 pp. 18mo. bound. (1862). *3s. 6d.*

As the Elements of Euclid are usually placed in the hands of young students, it is important to exhibit the work in such a form as will assist them in overcoming the difficulties which they experience on their first introduction to processes of continuous argument. No method appears to be so useful as that of breaking up the demonstrations into their constituent parts, and this plan has been adopted in the present edition. Each distinct assertion in the argument begins a new line; and at the end of the lines are placed the necessary references to the preceding principles on which the assertions depend. The longer propositions are distributed into subordinate parts, which are distinguished by breaks at the beginning of the lines. The Notes are intended to indicate and explain the principal difficulties, and to supply the most important inferences which can be drawn from the propositions. The work finishes with a collection of *Six hundred and twenty-five Exercises*, which have been selected principally from Cambridge Examination papers and have been tested by long experience. As far as possible they are arranged in order of difficulty. The Figures will be found to be large and distinct, and have been repeated when necessary, so that they always occur in immediate connexion with the corresponding te

ORKS by ISAAC TODHUNTER, M.A., F.R.S.—*continued.*

5.

e Integral Calculus

And its Applications.

With numerous Examples.

ond Edition. 342 pp. (1862).

Crown 8vo. cloth. 10*s.* 6*d.*

writing the present *Treatise on the* *ral Calculus,* the object has been to uce a work at once elementary and lete—adapted for the use of beginners, sufficient for the wants of advanced nts. In the selection of the propo- ns, and in the mode of establishing , the author has endeavoured to ex- fully and clearly the principles of ubject, and to illustrate all their most rtant results. In order that the stu- may find in the volume all that he ires, a large collection of Examples exercise has been appended to the rent chapters.

Analytical Statics.

With numerous Examples.

ond Edition. 330 pp. (1858).

Crown 8vo. cloth. 10*s.* 6*d.*

this work will be found all the pro- ions which usually appear in treatises 'heoretical Statics. To the different ters Examples are appended, which been selected principally from the versity and College Examination Pa- ; these will furnish ample exercise in application of the principles of the ect.

7. EXAMPLES OF

nalytical Geometry ' Three Dimensions.

pp. (1858). Crn. 8vo. cloth. 4*s.*

collection of examples in illustration nalytical Geometry of Three Dimen- s has long been required both by *ents and teachers,* and the present *is published* with the view of sup- *g the want.*

8. The

Differential Calculus.

With numerous Examples.

Third Edition, 398 pp. (1860).

Crown 8vo. cloth, 10*s.* 6*d.*

This work is intended to exhibit a comprehensive view of the Differential Calculus on the method of Limits. In the more elementary portions, explanations have been given in considerable detail, with the hope that a reader who is without the assistance of a tutor may be enabled to acquire a competent acquaintance with the subject. More than one investigation of a theorem has been frequently given, because it is believed that the student derives advantage from viewing the same proposition under different aspects, and that in order to succeed in the examinations which he may have to undergo, he should be prepared for a considerable variety in the order of arranging the several branches of the subject, and for a corresponding variety in the mode of demonstration.

9. Plane Co-Ordinate Geometry

AS APPLIED TO THE STRAIGHT LINE AND THE CONIC SECTIONS.

With numerous Examples.

Third and Cheaper Edition.

Crn. 8vo. cl. 326 pp. (1862). 7*s.* 6*d.*

This *Treatise* exhibits the subject in a simple manner for the benefit of beginners, and at the same time includes in one volume all that students usually require. The Examples at the end of each chapter will, it is hoped, furnish sufficient exercise, as they have been carefully selected with the view of illustrating the most important points, and have been tested by repeated experience with pupils. In consequence of the demand for the work proving much greater than had been originally anticipated, a large number of copies of the Third Edition has been printed, and a considerable reductio effected in the price.

By ISAAC TODHUNTER, M.A.

10.

AN ELEMENTARY TREATISE ON THE

Theory of Equations.

With a Collection of Examples.

Crown 8vo. cloth. 279 pp. (1861). 7*s.* 6*d.*

This treatise contains all the propositions which are usually included in elementary treatises on the Theory of Equations, together with a collection of Examples for exercise. This work may in fact be regarded as a sequel to that on Algebra by the same writer, and accordingly the student has occasionally been referred to the treatise on Algebra for preliminary information on some topics here discussed. The work includes three chapters on Determinants.

11.

History of the Progress of the Calculus of Variations

During the Nineteenth Century.

8vo. cloth. 532 pp. (1861). 12*s.*

It is of importance that those who wish to cultivate any subject may be able to ascertain what results have already been obtained, and thus reserve their strength for difficulties which have not yet been conquered. The Author has endeavoured in this work to ascertain distinctly what has been effected in the Progress of the Calculus, and to form some estimate of the manner in which it has been effected.

12.

Algebra for Beginners.

18mo. cloth, 264 pp. (1863). 2*s.* 6*d*.

Great pains have been taken to render *this work intelligible* to young students *by the use of simple* language and by *copious explanations.* In accordance *with the recommendation* of teachers, the *examples for exercise* are very numerous.

By J. H. PRATT, M.A.

Archdeacon of Calcutta, late Fellow of Gonville and Caius College, Cambridge.

A Treatise on Attractions,

La Place's Functions, and the Figure of the Earth.

Second Edition. Crown 8vo. 126 pp. (1861). Cloth. 6*s.* 6*d.*

In the present Treatise the author has endeavoured to supply the want of a work on a subject of great importance and high interest—La Place's Coefficients and Functions and the calculation of the Figure of the Earth by means of his remarkable analysis. No student of the higher branches of Physical Astronomy should be ignorant of La Place's analysis and its result—"a calculus," says Airy, "the most singular in its nature and the most powerful in its application that has ever appeared."

Theory of Errors of Observations

And the Combination of Observations.

By G. B. AIRY, M.A.

Astronomer Royal.

103 pp. (1861). Crown 8vo. 6*s.* 6*d.*

In order to spare astronomers and observers in natural philosophy the confusion and loss of time which are produced by referring to the ordinary treatises embracing both branches of Probabilities, the author has thought it desirable to draw up this work, relating only to Errors of Observation, and to the rules derivable from the consideration of these Errors, for the Combination of the Results of Observations. The Author has thus also the advantage of entering somewhat more fully into several points of interest to the observer, than can possibly be done in a General Theory of Probabilities.

AN ELEMENTARY TREATISE ON

The Planetary Theory

WITH A COLLECTION OF PROBLEMS.

By C. H. H. CHEYNE, B.A.

Scholar of St. John's College, Cambridge.

148 pp. 1862. Crn. 8vo. cloth. 6*s.*

'ORGE BOOLE, D.C.L., F.R.S.
ɔr of Mathematics in the Queen's University, Ireland.

ɔrential Equations

(1859). Crn. 8vo. cloth. 14*s.*

Author has endeavoured in this to convey as complete an account resent state of knowledge on the of Differential Equations as was nt with the idea of a work in- primarily, for elementary instruc- 'he object has been first of all to e wants of those who had no pre- :quaintance with the subject, and , quite to disappoint others who eek for more advanced informa- 'he earlier sections of each chapter that kind of matter which has been thought suitable for the r, while the latter ones are devoted ɔ an account of recent discovery, e discussion of such deeper ques- principle as are likely to present ves to the reflective student in con- with the methods and processes :evious course.

The Calculus of nite Differences.

'. (1860). Crown 8vo. cloth. 10*s.* 6*d.*

is work particular attention has id to the connexion of the methods ɔse of the Differential Calculus—a on which in some instances in- tar more than a merely formal . The work is in some measure d as a sequel to the Author's *Trea- Differential Equations*, and it has mposed on the same plan.

ɔmentary Statics.

Rev. GEORGE RAWLINSON,
ɔr of Applied Sciences, Elphinstone Coll., Bombay.

by the Rev. E. STURGES. M.A.
ctor of Kencott, Oxfordshire.

p.) 1860. Crn. 8vo. cl. 4*s.* 6*d.*

work is published under the au- *of H. M.* Secretary of State for *use* in the Government Schools *ges* in India.

By P. G. TAIT, M.A., and
W. J. STEELE, B.A.
Late Fellows of St. Peter's Coll. Camb.

Dynamics of a Particle.

With numerous Examples.

304 pp. (1856). Cr. 8vo. cl. 10*s.* 6*d.*

In this Treatise will be found all the ordinary propositions connected with the Dynamics of Particles which can be conveniently deduced without the use of D'Alembert's Principles. Throughout the book will be found a number of illustrative Examples introduced in the text, and for the most part completely worked out; others, with occasional solutions or hints to assist the student are appended to each Chapter.

By the Rev. G. F. CHILDE, M.A.
Mathematical Professor in the South African College.

Singular Properties of the Ellipsoid

And Associated Surfaces of the Nth *Degree.*

152 pp. (1861). 8vo. boards. 10*s.* 6*d.*

As the title of this volume indicates, its object is to develope peculiarities in the Ellipsoid; and further, to establish analogous properties in unlimited congeneric series of which this remarkable surface is a constituent.

By J. B. PHEAR, M.A.
Fellow and late Mathematical Lecturer of Clare College.

Elementary Hydrostatics

With numerous Examples and Solutions.

Third Edition. 156 pp. (1863). Crown 8vo. cloth. 5*s.* 6*d.*

"An excellent Introductory Book. The definitions are very clear; the descriptions and explanations are sufficiently full and intelligible; the investigations are simple and scientific. The examples greatly enhance its value."—ENGLISH JOURNAL OF EDUCATION.

This Edition contains 147 Examples, a[nd] solutions to all these examples are g[iven] at the end of the book.

By Rev. S. PARKINSON, B.D.

Fellow and Prælector of St. John's Coll. Cambridge.

1. Elementary Treatise on Mechanics.

With a Collection of Examples.

Third Edition revised and enlarged, 374 pp. (1863). Crn. 8vo. cl. 9*s*. 6*d*.

The Author has endeavoured to render the present volume suitable as a Manual for the junior classes in Universities and the higher classes in Schools. With this object there have been included in it those portions of theoretical Mechanics which can be conveniently investigated without the Differential Calculus, and with one or two short exceptions the student is not presumed to require a knowledge of any branches of Mathematics beyond the elements of Algebra, Geometry, and Trigonometry. A collection of Problems and Examples has been added, chiefly taken from the Senate-House and College Examination Papers—which will be found useful as an exercise for the student. In the Second Edition several additional propositions have been incorporated in the work for the purpose of rendering it more complete, and the Collection of Examples and Problems has been largely increased.

2. A Treatise on Optics

304 pp. (1859). Crown 8vo. 10*s*. 6*d*.

A collection of Examples and Problems has been appended to this work which are sufficiently numerous and varied in character to afford useful exercise for the student: for the greater part of them recourse has been had to the Examination Papers set in the University and the several Colleges during the last twenty years.

Subjoined to the copious Table of Contents *the author has* ventured to indicate *an elementary course of reading* not unsuitable *for the requirements* of the First *Three Days in the* Cambridge Senate *House Examinations.*

By R. D. BEASL

Head Master of Grant

AN ELEMENTARY T

Plane Trigon

With a numerous (
Examples

106 pp. (1858), stron
cloth. 3*s*.

This Treatise is special
use in Schools. The choi
been chiefly guided by t
of the three days' Exami
bridge, with the exception
parts in logarithms, wh
omitted. About *Four hu*
have been added, mainly
the Examination Papers
years, and great pains h
to exclude from the body
which might dishearten
their difficulty.

By J. BROOK SM

St. John's College, C

Arithmetic in
and Prac

For Advanced

PART I. Crown 8vo.

This work forms the firs
tise on Arithmetic, in w
has endeavoured, from v
ciples, to explain in a full
manner all the important
subject.

The proofs have in all
in a form entirely arith
author does not think tha
to be had to Algebra until
proof has become hopeles
plexing.

At the end of every ch
amples have been work
in which the best pr
operation have been ca

By G. H. PUCKLE, M.A.
Principal of Windermere College.

Conic Sections and Algebraic Geometry.

With numerous Easy Examples Progressively arranged.

Second Edit. enlarged and improved, 264 pp. (1856). Crown 8vo. 7*s.* 6*d.*

This book has been written with special reference to those difficulties and misapprehensions which commonly beset the student when he commences. With this object in view, the earlier part of the subject has been dwelt on at length, and geometrical and numerical illustrations of the analysis have been introduced. The Examples appended to each section are mostly of an elementary description. The work will, it is hoped, be found to contain all that is required by the upper classes of schools and by the generality of students at the Universities.

By EDWARD JOHN ROUTH, M.A.
Fellow and Assistant Tutor of St. Peter's College, Cambridge.

Dynamics of a System of Rigid Bodies.

With numerous Examples.

336 pp. (1860). Crown 8vo. cloth. 10*s.* 6*d.*

CONTENTS: Chap. I. Of Moments of Inertia.—II. D'Alembert's Principle.—III. Motion about a Fixed Axis.—IV. Motion in Two Dimensions.—V. Motion of a Rigid Body in Three Dimensions.—VI. Motion of a Flexible String.—VII. Motion of a System of Rigid Bodies.—VIII. Of Impulsive Forces.—IX. Miscellaneous Examples.

The numerous Examples which will be found at the end of each chapter have been chiefly selected from the Examination Papers set in the University and Colleges of Cambridge during the last few years.

THE CAMBRIDGE COURSE OF

Elementary Natural Philosophy.

BEING THE PROPOSITIONS IN MECHANICS AND HYDROSTATICS

In which those persons who are not Candidates for Honours are examined for the Degree B.A.

ORIGINALLY COMPILED

By J. C. SNOWBALL, M.A.,
Late Fellow of St. John's College.

Fifth Edition, Revised and Enlarged, and adapted for the Middle-Class Examinations.

By THOMAS LUND, B.D.,
Late Fellow and Lecturer of St. John's College. Editor of Wood's Algebra. &c.

155 pp. New Edition (1864). Crown 8vo. cloth. 5*s.*

This work will be found suited to the wants, not only of University Students, but also of many others who require a short course of Mechanics and Hydrostatics, and especially of the Candidates at our Middle-Class Examinations.

By N. M. FERRERS, M.A.
Fellow and Mathematical Lecturer of Gonville and Caius College, Cambridge.

AN ELEMENTARY TREATISE ON

Trilinear Co-Ordinates

The Method of Reciprocal Polars, and the Theory of Projections.

154 pp. (1161). Cr. 8vo. cl. 6*s.* 6*d.*

The object of the Author in writing on this subject has mainly been to place it on a basis altogether independent of the ordinary Carte ian System, instead of regarding it as only a special form of abridg Notation. A short chapter on Deter nants has been introduced.

By J. C. SNOWBALL, M.A.
Late Fellow of St. John's Coll. Cambridge.

Plane and Spherical Trigonometry.

With the Construction and Use of Tables of Logarithms.

Tenth Edition. 240 pp. (1863). Crown 8vo. 7*s.* 6*d.*

In preparing a new edition, the proofs of some of the more important propositions have been rendered more strict and general; and a considerable addition of more than *Two hundred Examples*, taken principally from the questions in the Examinations of Colleges and the University, has been made to the collection of Examples and Problems for practice.

By W. H. DREW, M.A.
Second Master of Blackheath School.

Geometrical Treatise on Conic Sections.

With a copious Collection of Examples.

Third Edition. Crown 8vo. cloth. 4*s.* 6*d.*

In this work the subject of Conic Sections has been placed before the student in such a form that, it is hoped, after mastering the elements of Euclid, he may find it an easy and interesting continuation of his geometrical studies. With a view also of rendering the work a complete Manual of what is required at the Universities, there have been either embodied into the text, or inserted among the examples, every book work question, problem, and rider, which has been proposed in the Cambridge examinations up to the present time.

Solutions to the Problems in Drew's Conic Sections.

Crown 8vo. cloth. 4*s.* 6*d.*

Senate-House Mathematical Problems.

With Solutions.

1848-51. By FERRERS and JACKSON. 8vo. 15*s.* 6*d.*
1848-51. (RIDERS). By JAMESON. 8vo. 7*s.* 6*d.*
1854. By WALTON and MACKENZIE. 10*s.* 6*d.*
1857. By CAMPION and WALTON. 8vo. 8*s.* 6*d.*
1860. By ROUTH and WATSON. Crown 8vo. 7*s.* 6*d.*
1864. By WALTON and WILKINSON. 8vo. cloth. 10*s.* 6*d.*

The above books contain Problems and Examples which have been set in the Cambridge Senate-house Examinations at various periods during the last twelve years, together with Solutions of the same. The Solutions are in all cases given by the Examiners themselves or under their sanction.

By H. A. MORGAN, M.A.
Fellow of Jesus College, Cambridge.

A Collection of Mathematical Problems and Examples.

With Answers.

190 pp. (1858). Crown 8vo. 6*s.* 6*d.*

This book contains a number of problems, chiefly elementary, in the Mathematical subjects usually read at Cambridge. They have been selected from the papers set during late years at Jesus College. Very few of them are to be met with in other collections, and by far the larger number are due to some of the most distinguished Mathematicians in the University.

By C. TAYLOR, B.A.
Scholar of St. John's College, Cambridge.

Geometrical Conics.

Including Anharmonic Ratio and Projection. With numerous Examples.

222 pp. 1863. Crown 8vo. 7*s.* 6*d.*

This work contains elementary proofs of the principal properties of Conic Sections, together with chapters on Projection and Anharmonic Ratio.

A Treatise on Solid Geometry.

By PERCIVAL FROST, M.A.,
St. John's College, and
JOSEPH WOLSTENHOLME, M.A.,
Christ's Coll. Cambridge.

472 pp. 8vo. cloth. 18*s*. 1863.

The authors have endeavoured to present before students as comprehensive a view of the subject as possible. Intending as they have done to make the subject accessible, at least in the earlier portion, to all classes of students, they have endeavoured to explain fully all the processes which are most useful in dealing with ordinary theorems and problems, thus directing the student to the selection of methods which are best adapted to the exigencies of each problem. In the more difficult portions of the subject, they have considered themselves to be addressing a higher class of students; there they have tried to lay a good foundation on which to build, if any reader should wish to pursue the science beyond the limits to which the work extends.

By PERCIVAL FROST, M.A.
Late Fellow of St. John's College;
Mathematical Lecturer of King's College.

Newton's Principia

SECTIONS 1, 2, and 3.
WITH NOTES AND ILLUSTRATIONS.
ALSO A

Collection of Problems

Principally intended as Examples of Newton's Methods.

Second Edit. 288 pp. (1863). 10*s*. 6*d*.

The author's principal intention is to explain difficulties which may be encountered by the student on first reading the Principia, and to illustrate the advantages of a careful study of the methods employed by Newton, by showing the extent to which they may be applied in the solution of problems; he has also endeavoured to give assistance to the student who is engaged in the study of the higher branches of Mathematics, by representing in a geometrical form several of the processes employed in the Differential and Integral Calculus, and in the analytical investigations of Dynamics.

By JOHN E. B. MAYOR, M.A.
Fellow and Classical Lecturer of St. John's College, Cambridge.

1. Juvenal.

With English Notes and an Index. New and Revised Edition. *Nearly Ready.* Crown 8vo. cloth. 7*s*. 6*d*.

"A School edition of Juvenal, which, for really ripe scholarship, extensive acquaintance with Latin literature, and familiar knowledge of Continental criticism, ancient and modern, is unsurpassed, we do not say among English School-books, but among English editions generally."—EDINBURGH REVIEW.

2. Cicero's Second Philippic.

With English Notes.

168 pp. (1861). Fcp. 8vo. cloth. 5*s*.

The Text is that of Halm's 2nd edition, (Leipzig, Weidmann, 1858), with some corrections from Madvig's 4th Edition (Copenhagen, 1858). Halm's Introduction has been closely translated, with some additions. His notes have been curtailed, omitted, or enlarged, at discretion; passages to which he gives a bare reference, are for the most part printed at length; for the Greek extracts an English version has been substituted. A large body of notes, chiefly grammatical and historical, has been added from various sources. A list of books useful to the student of Cicero, a copious Argument, and an Index to the introduction and notes, complete the book.

The Chief Rules of Latin Syntax.

COMPILED BY
G. HALE PUCKLE, M.A.
Head Master of Windermere College.

Fcp. 8vo. 1*s*.

Rules for the Quantity of Syllables in Latin

Fcp. 8vo. 1*s*.

An Elementary Latin Grammar.

By H. J. ROBY, M.A.,

Under-Master of Dulwich College Upper School, late Fellow and Classical Lecturer of St. John's College, Cambridge.

18mo. 2*s.* 6*d.*

The Author's experience in practical teaching has induced an attempt to treat Latin Grammar in a more precise and intelligible way than has been usual in school books. The facts have been derived from the best authorities, especially Madvig's Grammar and other works. The works also of Lachmann, Ritschl, Key, and others have been consulted on special points. The accidence and prosody have been simplified and restricted to what is really required by boys. In the Syntax an analysis of sentences has been given, and the uses of the different cases, tenses and moods briefly but carefully described. Particular attention has been paid to a classification of the uses of the subjunctive mood, to the prepositions, the *oratio obliqua*, and such sentences as are introduced by the English '*that.*' Appendices treat of the Latin forms of Greek nouns, abbreviations, dates, money, &c. The Grammar is written in English.

ELEMENTARY HISTORY OF THE

Book of Common Prayer.

For the Use of Schools and popular reading.

By FRANCIS PROCTER, M.A.,

Vicar of Witton, Norfolk, late Fellow of St. Catharine's College, Cambridge.

18mo. *cloth.* 2*s.* 6*d.*

The Author having been frequently urged to give a popular abridgement of his larger work in a form which should be suited for use in Schools and for general readers, has attempted in this book to trace the History of the Prayer-Book, and to supply to the English reader the general *results which* in the larger work are *accompanied by elaborate* discussions and *references to authorities* indispensable to *the student. It is hoped that this book may form a useful manual to assist* people *generally to a more intelligent* use of the *forms of our Common Prayer.*

By B. DRAKE, M.A.

Late Fellow of King's Coll. Cambridge.

1. Demosthenes on the Crown.

With English Notes.

Second Edition. To which is prefixed ÆSCHINES AGAINST CTESIPHON. With English Notes.

287 pp. (1860). Fcap. 8vo. cl. 5*s.*

The first edition of the late Mr. Drake's edition of Demosthenes de Corona having met with considerable acceptance in various Schools, and a new edition being called for, the Oration of Æschines against Ctesiphon, in accordance with the wishes of many teachers, has been appended with useful notes by a competent scholar.

2. Æschyli Eumenides

With English Verse Translation, Copious Introduction, and Notes.

8vo. 144. pp. (1853). 7*s.* 6*d.*

"Mr. Drake's ability as a critical Scholar is known and admitted. In the edition of the Eumenides before us we meet with him also in the capacity of a Poet and Historical Essayist. The translation is flowing and melodious, elegant and scholarlike. The Greek Text is well printed: the notes are clear and useful."—GUARDIAN.

By C. MERIVALE, B.D.

Author of "History of Rome," &c.

Sallust.

With English Notes.

Second Edition. 172 pp. (1858). Fcap. 8vo. 4*s.* 6*d.*

"This School edition of Sallust is precisely what the School edition of a Latin author ought to be. No useless words are spent in it, and no words that could be of use are spared. The text has been carefully collated with the best editions. With the work is given a full current of extremely well-selected annotations."—THE EXAMINER.

The "CATILINA" *and* "JUGURTHA" *may be had separately, price* 2*s.* 6*d.* *each, bound in cloth.*

By the Rev. J. WRIGHT, M.A.
Of Trinity College, Cambridge.

1. Help to Latin Grammar.

With Easy Exercises, and Vocabulary.

Crown 8vo. cloth. 4*s.* 6*d.*

Never was there a better aid offered alike to teacher and scholar in that arduous pass. The style is at once familiar and strikingly simple and lucid; and the explanations precisely hit the difficulties, and thoroughly explain them."—English Journal of Education.

2. Hellenica.

A FIRST GREEK READING BOOK.

Second Edition. With Vocabulary. Fcap. 8vo. cl. 3*s.* 6*d.*

In the last twenty chapters of this volume, Thucydides sketches the rise and progress of the Athenian Empire in so clear a style and in such simple language, that the author doubts whether any easier or more instructive passages can be selected for the use of the pupil who is commencing Greek.

3. The Seven Kings of Rome.

A First Latin Reading Book.

Third Edit. Fcap. 8vo. cloth. 3*s.*

This work is intended to supply the pupil with an easy Construing-book, which may, at the same time, be made the vehicle for instructing him in the rules of grammar and principles of composition. Here Livy tells his own pleasant stories in his own pleasant words. Let Livy be the master to teach a boy Latin, not some English collector of sentences, and he will not be found a dull one.

4. Vocabulary and Exercises on "The Seven Kings of Rome."

Fcp. 8vo. cloth. 2*s.* 6*d.*

*** *The Vocabulary and Exercises may also be had bound up with "The Seven Kings of Rome."* 5*s.* cloth.

By EDWARD THRING, M.A.
Head Master of Uppingham School.

Elements of Grammar Taught in English.

With Questions.

Third Edition. 136 pp. (1860). Demy 18mo. 2*s.*

2. The Child's English Grammar.

New Edition. 86 pp. (1859). Demy limp cloth, 18mo. 1*s.*

The Author's effort in these two books has been to point out the broad, beaten, every-day path, carefully avoiding digressions into the byeways and eccentricities of language. This Work took its rise from questionings in National Schools, and the whole of the first part is merely the writing out in order the answers to questions which have been used already with success. Its success, not only in National Schools, from practical work in which it took its rise, but also in classical schools, is full of encouragement.

3. A Latin Gradual.

A First Latin Construing Book for Beginners.

167 pp. 1864. 18mo. cloth. 2*s.* 6*d.*

The main plan of this little work has been well tested: experience however has led to considerable changes in the way of working this out.

The intention is to supply by easy steps a knowledge of Grammar, combined with a good vocabulary; in a word, a book which will not require to be forgotten again as the learner advances.

A short practical manual of common Mood constructions, with their English equivalents form the second part.

4. School Songs.

A COLLECTION OF SONGS FOR SCHOOLS.

WITH THE MUSIC ARRANGED FOR FOUR VOICES.

Edited by Rev. E. THRING and H. RICCIUS.

Music Size. 7*s.* 6*d.*

By EDWARD THRING, M.A.

5. A First Latin Construing Book.

104 pp. (1855). Fcap. 8vo. 2*s*. 6*d*.

This Construing Book is drawn up on the same sort of graduated scale as the Author's *English Grammar*. Passages out of the best Latin Poets are gradually built up into their perfect shape. The few words altered, or inserted as the passages go on, are printed in Italics. It is hoped by this plan that the learner, whilst acquiring the rudiments of language, may store his mind with good poetry and a good vocabulary.

By C. J. VAUGHAN, D.D.

Vicar of Doncaster, and Chaplain in Ordinary to the Queen.

St. Paul's Epistle to the Romans.

The Greek Text with English Notes.

Second Edition. Crown 8vo. cloth. (1861). 5*s*.

By dedicating this work to his elder Pupils at Harrow, the Author hopes that he sufficiently indicates what is and what is not to be looked for in it. He desires to record his impression, derived from the experience of many years, that the Epistles of the New Testament, no less than the Gospels, are capable of furnishing useful and solid instruction to the highest classes of our Public Schools. If they are taught accurately, not controversially; positively, not negatively; authoritatively, yet not dogmatically; taught with close and constant reference to their literal meaning, to the connexion of their parts, to the sequence of their argument, as well as to their moral and spiritual instruction; they will interest, they will inform, they will elevate; they will inspire a reverence for Scripture never to be disregarded, they will awaken a desire to drink more deeply of the Word of God, certain hereafter to be gratified and fulfilled.

Notes for Lectures On Confirmation.

With Suitable Prayers.

By C. J. VAUGHAN, D.D.

6th Edition. 70 pp. (1864). Fcp. 8vo. 1*s*. 6*d*.

This work, originally prepared for the use of Harrow School, is published in the belief that it may assist the labours of those who are engaged in preparing candidates for Confirmation, and who find it difficult to lay their hand upon any one book of suitable instruction at once sufficiently full to furnish a synopsis of the subject, and sufficiently elastic to give free scope to the individual judgment in the use of it. It will also be found a handbook for those who are being prepared, as presenting in a compact form the very points which a lecturer would wish his hearers to remember.

The Church Catechism Illustrated and Explained. By ARTHUR RAMSAY, M.A. Second Edition. 18mo. 1*s*. 6*d*.

Hand-Book to Butler's Analogy. By C. A. SWAINSON, M.A. 55 pp. (1856). Crown 8vo. 1*s*. 6*d*.

History of the Christian Church during the First Three Centuries, and the Reformation in England. By W. SIMPSON, M.A. Fourth Edition. Fcp. 8vo. cloth. 3*s*. 6*d*.

Analysis of Paley's Evidences of Christianity. By CHARLES H. CROSSE, M.A. 115 pp. (1855). 18mo. 3*s*. 6*d*.

CAMBRIDGE MANUALS

FOR THEOLOGICAL STUDENTS.

This Series of THEOLOGICAL MANUALS has been published with the aim of supplying Books concise, comprehensive, and accurate; convenient for the Student, and yet interesting to the general reader.

1. History of the Christian Church during the Middle Ages. By ARCHDEACON HARDWICK. Second Edition. 482 pp. (1861). With Maps. Crown 8vo. cloth. 10*s.* 6*d.*

This Volume claims to be regarded as an integral and independent treatise on the Mediæval Church. The History commences with the time of Gregory the Great, to the year 1520,—the year when Luther, having been extruded from those Churches that adhered to the Communion of the Pope, established a provisional form of government and opened a fresh era in the history of Europe.

2. History of the Christian Church during the Reformation. By ARCHDN. HARDWICK. 459 pp. (1856). Crown 8vo. cloth. 10*s.* 6*d.*

This Work forms a Sequel to the Author's Book on The Middle Ages. The Author's wish has been to give the reader a trustworthy version of those stirring incidents which mark the Reformation period.

3. History of the Book of Common Prayer. With a Rationale of its Offices. By FRANCIS PROCTER, M.A. Fifth Edition. 464 pp. (1860). Crown 8vo. cloth. 10*s.* 6*d.*

In the course of the last twenty years *the whole* question of liturgical knowledge has been reopened with great learning and accurate research, and it is mainly with the view of epitomizing their extensive publications, and correcting by their help the errors and misconceptions which had obtained currency, that the present volume has been put together.

4. History of the Canon of the New Testament during the First Four Centuries. By BROOKE FOSS WESTCOTT, M.A. 594 pp. (1855). Crown 8vo. cloth. 12*s.* 6*d.*

The Author has endeavoured to connect the history of the New Testament Canon with the growth and consolidation of the Church, and to point out the relation existing between the amount of evidence for the authenticity of its component parts and the whole mass of Christian literature. Such a method of inquiry will convey both the truest notion of the connexion of the written Word with the living Body of Christ, and the surest conviction of its divine authority.

5. Introduction to the Study of the GOSPELS. By BROOKE FOSS WESTCOTT, M.A. 458 pp. (1860). Crown 8vo. cloth. 10*s.* 6*d.*

This book is intended to be an Introduction to the *Study* of the Gospels. In a subject which involves so vast a literatur much must have been overlooked; but t Author has made it a point at leas study the researches of the great wr and consciously to neglect none.

Uniformly printed in 18mo. with Vignette Titles by T. Woolner, W. Holman Hunt, J. Noel Paton, &c.

Handsomely bound in extra cloth, 4s. 6d. Morocco plain, 7s. 6d. Morocco extra, 10s. 6d. each Volume.

1. THE GOLDEN TREASURY

OF THE BEST SONGS AND LYRICAL POEMS IN THE ENGLISH LANGUAGE.

Selected and arranged, with Notes, by F. T. PALGRAVE.

2. THE FAIRY BOOK.

THE BEST POPULAR FAIRY STORIES, SELECTED AND RENDERED ANEW

By the Author of "John Halifax."

3. THE CHILDREN'S GARLAND.

FROM THE BEST POETS.

Selected and Arranged by COVENTRY PATMORE.

4. THE PILGRIM'S PROGRESS

FROM THIS WORLD TO THAT WHICH IS TO COME.

By JOHN BUNYAN.

Large-paper copies, crown 8vo. 7s. 6d.; or bound in half morocco, 10s. 6d.

5. THE BOOK OF PRAISE.

FROM THE BEST ENGLISH HYMN WRITERS.

Selected and arranged by ROUNDELL PALMER.

*** ROYAL EDITION, 6s. Morocco Extra, 12s. 6d. LARGE-TYPE EDITION, demy 8vo. handsomely printed and bound, 10s. 6d.; or best Levant morocco by Riviere, 28s.

6. BACON'S ESSAYS AND COLOURS OF GOOD AND EVIL.

With Notes and Glossarial Index, by W. ALDIS WRIGHT, M.A., Trinity College, Cambridge.

*** Large Paper Copies, Crown 8vo. 7s. 6d., or bound in half-morocco, 10s. 6d.

7. THE JEST BOOK.

THE CHOICEST ANECDOTES AND SAYINGS

Selected and Arranged by MARK LEMON.

With a Vignette by Charles Kean.

8. THE BALLAD BOOK.

A SELECTION OF THE CHOICEST BRITISH BALLADS.

Edited by WILLIAM ALLINGHAM.

With a Vignette by J. Noel Paton, R.S.A.

Jonathan Palmer, Printer, Cambridge.

ELEMENTARY SCHOOL CLASS BOOKS.

The volumes of this Series of ELEMENTARY SCHOOL CLASS BOOKS are handsomely printed in a form that, it is hoped, will assist the young student as much as clearness of type and distinctness of arrangement can effect. They are published at a moderate price to ensure an extensive sale in the Schools of the United Kingdom and the Colonies.

A BRIEF BIOGRAPHICAL DICTIONARY. Compiled and arranged by CHARLES HOLE, B.A. Trinity College, Cambridge. Pot 8vo. (the same size as the Golden Treasury Series), neatly and strongly bound in cloth, price 4s. *6d.*

The publishers venture to believe that this little comprehensive work will become as indispensable to all English readers as an English Dictionary.

EUCLID FOR COLLEGES AND SCHOOLS. By I. TODHUNTER, M.A., F.R.S., Fellow and Principal Mathematical Lecturer of St John's College, Cambridge. 18mo. 3s. *6d.*

ALGEBRA FOR BEGINNERS. By I. TODHUNTER, M.A., F.R.S. 18mo. 2s. *6d.*

⁂ A KEY to this work will shortly be published.

THE SCHOOL CLASS BOOK OF ARITHMETIC. By BARNARD SMITH, M.A., late Fellow of St Peter's College, Cambridge. Parts I. and II. are now ready. 18mo. limp cloth, price 10*d.* each.

AN ELEMENTARY LATIN GRAMMAR. By H. J. ROBY, M.A., Under Master of Dulwich College Upper School, late Fellow and Classical Lecturer of St John's College, Cambridge. 18mo. 2s. *6d.*

MYTHOLOGY FOR LATIN VERSIFICATION. A Brief Sketch of the Fables of the Ancients, prepared to be rendered into Latin Verse for Schools. By F. HODGSON, B.D., late Provost of Eton College. *New Edition.* Revised by F. C. HODGSON, M.A., Fellow of King's College, Cambridge. 18mo. 3s.

A LATIN GRADUAL FOR BEGINNERS. First Latin Construing Book. By EDWARD THRING, M Head Master of Uppingham School. 18mo. *2s. 6d.*

SHAKESPEARE'S TEMPEST. The Text tal from "The Cambridge Shakespeare." With Gloss and Explanatory Notes. By the Rev. J. M. JEPHS 18mo. cloth. *3s. 6d.*

LESSONS IN ELEMENTARY BOTANY. T Part on Systematic Botany based upon Material left Manuscript by the late Professor HENSLOW. With ne Two Hundred Illustrations. By DANIEL OLIVER, F.R F.L.S., Keeper of the Herbarium and Library of the R Gardens, Kew, and Professor of Botany in University (lege, London. 18mo. cloth. *4s. 6d.*

AN ELEMENTARY HISTORY OF THE BO(OF COMMON PRAYER. By FRANCIS PROCTER, M. Vicar of Witton, Norfolk, late Fellow of St Cathari College, Cambridge. 18mo. *2s. 6d.*

THE BIBLE IN THE CHURCH. A Popu Account of the Collection and Reception of the H Scriptures in the Christian Churches. By BROOKE F WESTCOTT, M.A. 18mo. *4s. 6d.*

LESSONS IN ELEMENTARY PHYSIOLOG With numerous Illustrations. By T. H. HUXLEY, F.R Professor of Natural History in the Government School Mines. [*Nearly Ready,* in 18

THE BIBLE WORD-BOOK. A Glossary of (English Bible Words. By J. EASTWOOD, M.A., of John's College, and W. ALDIS WRIGHT, M.A., Trinity (lege, Cambridge. [*In the Pr*

POPULAR ASTRONOMY: A Series of Lectu delivered at Ipswich. By GEORGE BIDDELL AIRY, As nomer Royal. [*In the Pr*